Application of a new composite BIE for 3-D acoustic problems

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Abstract

Applications of a new and efficient composite BIE formulation for 3-D acoustic problems are studied in this paper, based on an improved weakly-singular form of the hypersingular boundary integral equation (HBIE). This new form of the HBIE involves only tangential derivatives of the density function and thus its discretization using the boundary element method (BEM) is easier to perform. C^0 continuous (conforming) quadratic elements are employed in the discretization of this weakly-singular form of the HBIE, as compared with using nonconforming and/or C^1 continuous boundary elements which were advocated earlier. Numerical examples of both scattering and radiation problems with various geometries are presented in this paper to demonstrate the accuracy and versatility of this improved composite BIE for 3-D acoustics.

1 Introduction

The linear combination of the conventional boundary integral equation (CBIE) and hypersingular boundary integral equation (HBIE) was first introduced by Burton and Miller [1] in 1971 for 3-D exterior acoustic problems. This composite BIE formulation has been demonstrated to be the most effective and theoretically-sound approach among all the methods available in dealing with the fictitious-eigenfrequency difficulty (FED) in acoustics. The issue of improving the efficiency of the composite BIE has been focused on how to tackle the hypersingular integrals. Many regularized or weakly-singular forms of HBIE have been reported in the literature, see, e.g., [2-4]. Although the discretization of these regularized forms still demands the use of boundary elements with C^1 continuity near each node, as demanded by the theory [5], many successful studies using C^0 conforming elements, on acoustic as well as elastostatic problems, have been reported [2-4, 6]. A recent development of this relaxation of the smoothness requirement can be found in [7].

In this paper, the effectiveness and efficiency of a new weakly-singular form of the HBIE for 3-D acoustic wave problems presented in [8] is further studied. Compared with the weakly-singular form of the HBIE for acoustic problems published earlier in [4], this new form involves only tangential derivatives of the density function, and thus its discretization using the BEM is easier to perform. Instead of using nonconforming and C^1 continuous boundary elements as advocated in [4], C^0 conforming quadratic elements are employed in the discretization of this new weakly-singular form of the HBIE. The new form of the HBIE is applied in the composite BIE formulation to overcome the fictitious eigenfrequency difficulties in 3-D acoustics using BIEs. Numerical examples of both scattering and radiation problems are given to demonstrate the accuracy and versatility of the new weakly-singular form of the HBIE. Report on the test of an iterative solver is also presented.

2 The new weakly-singular form of the HBIE

The conventional boundary integral equation (CBIE) for acoustic problem can be written as:

$$C(P_o)\phi(P_o) = \int_S \left[G(P,P_o)\frac{\partial \phi(P)}{\partial n} - \frac{\partial G(P,P_o)}{\partial n}\phi(P) \right] dS(P) + \phi^I(P_o), \tag{1}$$

where ϕ is the total acoustic wave satisfying the Helmholtz equation for time harmonic waves, ϕ^I is a prescribed incident wave, $G(P,P_o) = e^{ikr}/(4\pi r)$ is the full space Green's function for the Helmholtz equation, and $C(P_o)$ = 1/2 when the boundary S is smooth. Equation (1) is the singular form of CBIE and can be readily converted to its weakly-singular form.

The hypersingular BIE (HBIE), which is the derivative of the CBIE, can be converted into a weakly-singular form by employing a two-term Taylor's series subtraction from the density function and using the identities for the Green's function [9] to evaluate the added-back terms [4]. In [8], the tangential gradient, instead of the total gradient of ϕ was used and found to be sufficient to remove or regularize the hypersingularity of the kernel. Thereby an improved weakly-singular form was found as following [8]:

$$
\begin{aligned}
&\frac{\partial\phi(P_o)}{\partial n_o} + \int_S \frac{\partial^2 \overline{G}(P,P_o)}{\partial n \partial n_o}\left[\phi(P) - \phi(P_o) - \frac{\partial\phi(P_o)}{\partial\xi_\alpha}(\xi_\alpha - \xi_{o\alpha})\right] dS(P) \\
&+ \int_S \frac{\partial^2}{\partial n \partial n_o}\left[\; G(P,P_o) - \overline{G}(P,P_o)\right]\phi(p)\, dS(P) \\
&+ e_{\alpha k}\frac{\partial\phi(P_o)}{\partial\xi_\alpha}\int_S \left[\frac{\partial\overline{G}(P,P_o)}{\partial n_o} n_k(P) + \frac{\partial\overline{G}(P,P_o)}{\partial n} n_k(P_o)\right] dS(P) \\
&= \int_S \left[\frac{\partial G(P,P_o)}{\partial n_o} + \frac{\partial\overline{G}(P,P_o)}{\partial n}\right]\frac{\partial\phi(P)}{\partial n}\, dS(P) \\
&- \int_S \frac{\partial\overline{G}(P,P_o)}{\partial n}\left[\frac{\partial\phi(P)}{\partial n} - \frac{\partial\phi(P_o)}{\partial n}\right] dS(P) + \frac{\partial\phi^I(P_o)}{\partial n_o}, \qquad \forall P_o \in S, \qquad (2)
\end{aligned}
$$

where ξ_1 and ξ_2 are the first two (tangential) coordinates of a local curvilinear coordinate system $O\xi_1\xi_2\xi_3$ with origin at point P_o (Fig. 1), $e_{\alpha k} = \partial\xi_\alpha / \partial x_k$ (k = 1, 2, 3) are the first two column vectors of the inverse of the Jacobian matrix and $(\xi_\alpha - \xi_{o\alpha}) = e_{\alpha k}(x_k - x_{ok})$.

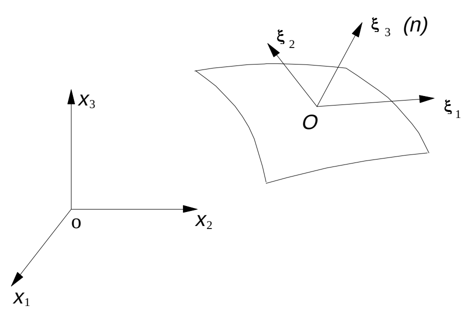

Figure 1: The global and local coordinate systems.

Equation (2) is the desired weakly-singular form of the hypersingular BIE for acoustic wave problems. It is interesting to note that the HBIE for acoustics in the form of Eqn (2) exhibits a term-by-term correspondence with the HBIE for elastodynamics developed in [10]. Compared with the form used earlier [4], this new form is much easier to discretize because the tangential derivatives of $\phi(P)$ can be evaluated readily using shape functions on an element. The discretization procedure for Eqn (2) is similar to that described in [4].

3 Numerical examples

Studies on the scattering and radiation from bodies of spherical, cylindrical and submarine-like shapes were conducted to verify the developed composite BIE with the conforming quadratic elements.

The first numerical study is for a spherical body immersed in an acoustic medium, for which analytical solutions [11] are available for both the scattering and radiation problems.

Figure 2 shows the radiated waves when the sphere is pulsating with

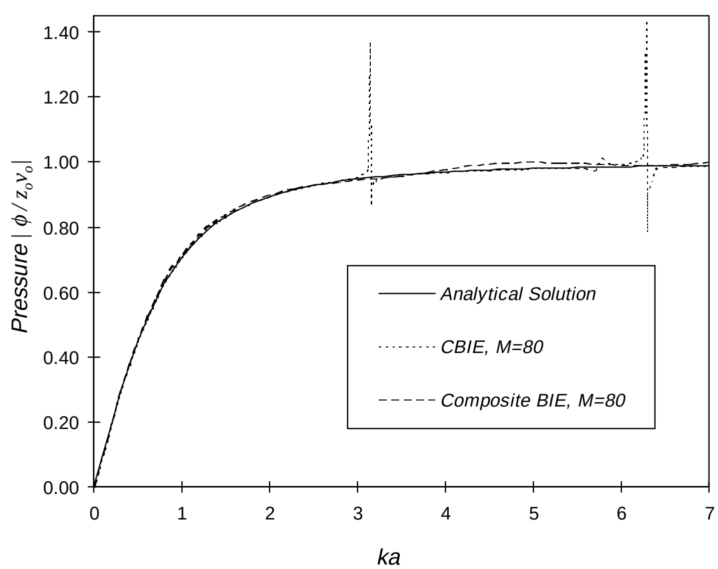

Figure 2: Pressure on the surface of the pulsating sphere.

a uniform radial velocity v_o on the surface *S* and $\partial\phi/\partial n = ikz_o v_o$, with z_o being the characteristic impedance. 80 boundary elements were used. At $ka = \pi$ and 2π the fictitious eigenfrequencies for the CBIE can be clearly identified (near which the CBIE results deviate substantially from the analytical solution). The composite BIE, however, provides very satisfactory and stable results throughout the range of the frequencies.

The second study is for a capsule-like cylindrical body. Since no analytical solutions are readily available for this problem, the commercial boundary element software *COMET/Acoustics* is employed in the verification for the radiation problem. The same mesh with 216 elements and 626 nodes (Fig. 3) is used for both *COMET/Acoustics* and the developed composite BIE code. Fig. 4 shows the study of radiated wave for a pulsating capsule. Fig. 5 shows the back scattering wave of the capsule impinged upon by an incident wave ϕ^I in the *x*-direction. Four fictitious eigenfrequencies of the CBIE were identified by monitoring the condition number of the system of equations at each frequency and the stability in the CBIE results. It can be clearly seen that both the results using *COMET* direct BIE and the CBIE deteriorate near the four fictitious eigenfrequencies. The composite BIE provided stable and smooth results throughout the frequency range, with very low condition numbers observed. The results from *COMET* CHIEF [12] method stayed closely along the same smooth curve although suitable CHIEF points were required.

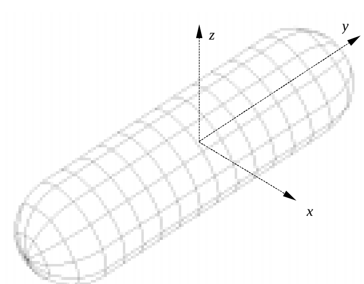

Figure 3: A cylindrical (capsule-like) body with radius = 1.0m and total length = 7.0m.

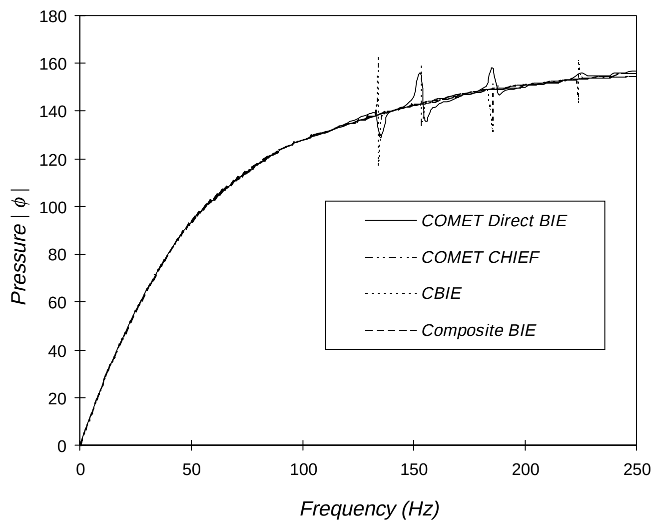

Figure 4: Radiated wave from the cylinder in the lateral direction.

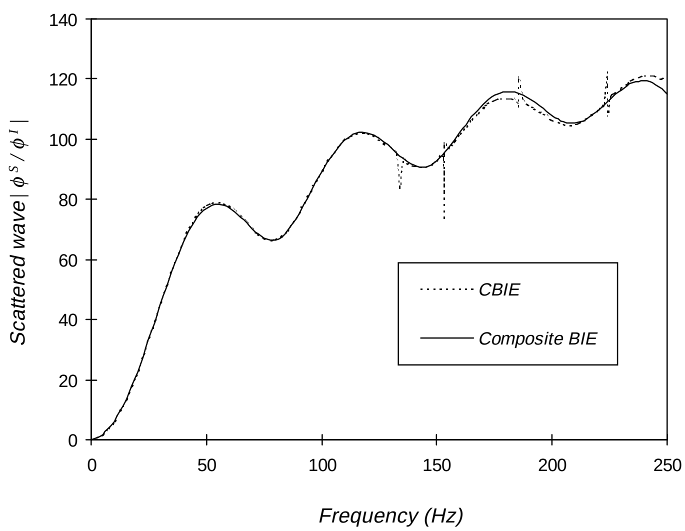

Figure 5: Backscattering from the cylinder (with side incident wave).

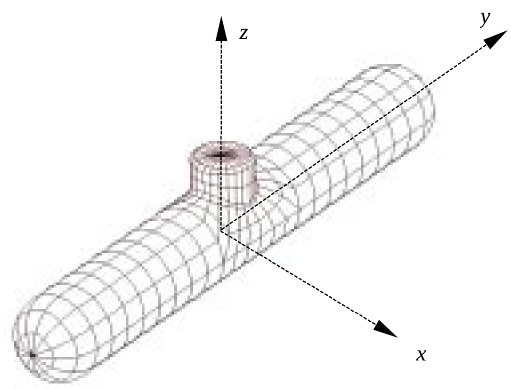

Figure 6: A generic submarine model with main radius = 5m and total length = 70m.

The third example is a generic submarine submerged in water. The same mesh with 492 elements and 1430 nodes (Fig. 6) was used for both CBIE and the composite BIE code. The scattered wave for the submarine impinged upon by an incident wave in the x_direction was studied. The back scattering and forward scattering waves are shown in Fig. 7 and Fig. 8, respectively. The fictitious-eigenfrequency difficulty is clearly shown in the CBIE case and completely removed in the composite BIE case. Small deviation between results of CBIE and composite BIE can be observed. This may be due to the complexity of the kernel function of composite BIE and can be resolved by increasing mesh density.

An iterative solver [13] is being tested for solving the linear system of equations, and compared with the direct solver. Dramatic speedup in solution time, with a factor as high as 52 times faster, is observed, except for the composite BIE case of the generic submarine, in which special pre-conditioner may be needed to regularize the linear equation systems. Detailed results will be reported at the conference.

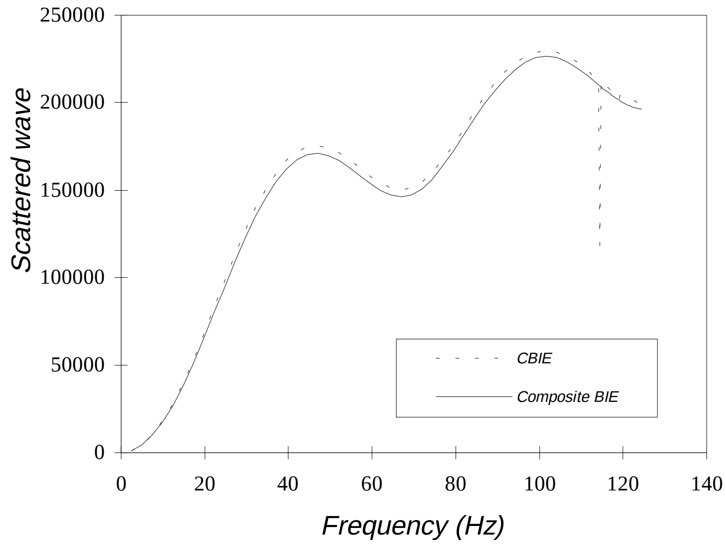

Figure 7: Back scattering from the submarine at point (-175, 0, 0).

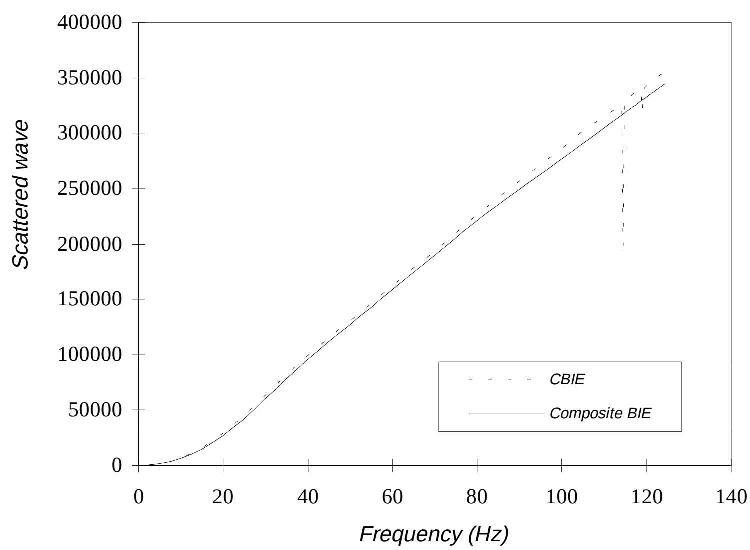

Figure 8: Forward scattering from the submarine at point (175, 0, 0).

4 Conclusion

The effectiveness and efficiency of the new weakly-singular form of the hypersingular boundary integral equation for 3-D acoustic wave problems is shown by numerical examples of radiation and scattering problems for spherical, cylindrical and submarine-like objects. Conforming C^0 quadratic boundary elements are employed in the discretization as a relaxation of the theoretical smoothness requirement and satisfactory results are retained. An iterative solver is tested with dramatic speedup being observed for most of the cases.

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References

[1] Burton, A. J. & Miller, G. F. The application of integral equation methods to the numerical solution of some exterior boundary-value problems, *Proc. R. Soc. Lond. A*, **323**, pp. 201-210, 1971.

[2] Chien, C. C., Rajiyah, H. and Atluri, S. N. An effective method for solving the hypersingular integral equations in 3-D acoustics, *J. Acoust. Soc. Am.*, **88**, 918-937, 1990.

[3] Wu, T. W., Seybert, A. F. & Wan, G. C. On the numerical implementation of a Cauchy principal value integral to insure a unique solution for acoustic radiation and scattering, *J. Acoust. Soc. Am.*, **90**, pp. 554-560, 1991.

[4] Liu, Y. J. & Rizzo, F. J. A weakly-singular form of the hypersingular boundary integral equation applied to 3-D acoustic wave problems, *Comput. Methods Appl. Mech. Engrg.*, **96**, pp. 271-287, 1992.

[5] Krishnasamy, G., Rizzo, F. J. & Rudolphi, T. J. Continuity requirements for density functions in the boundary integral equation method, *Comput. Mech.*, **9**, pp. 267-284, 1992.

[6] Cruse, T. A. & Richardson, J. D. Non-singular Somigliana stress identities in elasticity, *Int. J. Numer. Methods Engrg.*, **39**, pp. 3273-3304, 1996.

[7] Martin, P.A., Rizzo, F. J. & Cruse, T. A. Smoothness-relaxation strategies for singular and hypersingular integral equations, *Int. J. Numer. Methods Engrg*., in press.

[8] Liu, Y. J. & Chen, S. A new form of the hypersingular boundary integral equation for 3-D acoustics and its implementation with C^0 boundary elements, *Comput. Methods Appl. Mech. Engrg*., in review, 1997.

[9] Liu, Y. J. & Rudolphi, T. J. Some identities for fundamental solutions and their applications to weakly-singular boundary element formulations, *Engrg. Anal. Boundary Elements*, **8**, pp. 301-311, 1991.

[10] Liu, Y. J. & Rizzo, F. J. Hypersingular boundary integral equations for radiation and scattering of elastic waves in three dimensions, *Comput. Methods Appl. Mech. Engrg.*, **107**, pp. 131-144, 1993.

[11] Skudrzyk, E. *The Foundation of Acoustics*, New York, Springer-Verlag Vol. Chap. 20, 1971.

[12] Seybert, A. F. & Rengarajan, T. K. The use of chief to obtain unique solutions for acoustic radiation using boundary integral equations, *J. Acoust. Soc. Am.*, **81**, pp. 1299-1306, 1987.

[13] Freund, R. W. & Nachtigal, N. M. QMRPACK: a package of QMR algorithms, *ACM Transactions on Mathematical Software*, **22**, pp. 46-77, 1996.