Elastic stability analysis of thin plate by the boundary element method – a new formulation

YIJUN LIU

PO Box 118, Department of Aircraft Engineering, Northwestern Polytechnical University, Xian, Shaanxi, People's Republic of China

A new boundary integral equation formulation for the elastic stability analysis of thin plate is presented in this paper. This formulation involves only two kinds of integral equations which are similar to those employed in the linear analysis of plate bending problems by the boundary element method and are suitable to plates with arbitrary plan forms and under general boundary conditions or in-plane load conditions. A new simple boundary element discretisation scheme is established based on these integral equations. Satisfactory numerical results obtained on a microcomputer with constant elements clearly show the applicability and efficiency of the approach developed in this paper.

Key Words: elastic stability, thin plate, BEM.

INTRODUCTION

The boundary element method (BEM), as a new and powerful tool of numerical analysis, has aroused the wide attention of engineers and research workers in various fields. Its application to the linear analysis of plate bending problems has been investigated intensively in several papers.[1–18] The subjects of free vibration,[11] time-dependent inelastic analysis of transverse deflection,[19] and Reissner's plate model[20] have also been dealt with for plate bending problems. In the field of finite deflection analysis of thin elastic plate, much progress has been made in the past few years.[21–27] For the elastic stability analysis of thin plate by BEM, however, only two papers have been published according to the best knowledge of the author. Gospodinov and Ljutskanov[11] presented an indirect formulation under the condition that the plate is loaded with uniformly distributed normal forces p (i.e. $\sigma_x = \sigma_y =$ constant, $\tau_{xy} = 0$ in the domain). Costa and Brebbia[28] developed a direct BEM formulation, with three kinds of integral equations (five equations altogether), for the buckling problem of plates under general in-plan loading and boundary conditions. The numerical results provided by Costa and Brebbia showed the applicability and potentiality of BEM for the plate stability analysis.

Based on the previous work,[25–27] this paper presents a new boundary integral equation formulation for the elastic stability analysis of thin plate. This formulation needs only two kinds of integral equations (three equations altogether) which are similar to those employed in the linear plate bending problems and can be applied to plates with arbitary plan forms and under arbitrary in-plan load and boundary conditions. A much simplified direct BEM formulation, with the above mentioned integral equations, is developed with constant elements. The dimensions of matrices in this BEM discretisation can be greatly reduced for plates with clamped and simply supported edges or their combination, which makes the stability analysis of plates by BEM more efficient and practical. Numerical examples of square and circular plates under various load and boundary conditions were studied on a microcomputer, and the results clearly demonstrated the accuracy and efficiency of the approach developed in this paper.

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BASIC RELATIONSHIPS

The co-ordinate system $0x_1x_2z$ (i.e. $0xyz$) and notations of some basic quantities are shown in Fig. 1a. Let E be the elastic modulus of the material, ν the Poisson's ratio, h the plate thickness and $D = Eh^3/12(1-\nu^2)$ the bending rigidity. For concise, the Einstein summation convention is implied and the range of values of subscripts $i, j, k, l, \ldots$ is from 1 to 2.

The relationships of bending, twisting moments M_{ij} and deflection w can be represented as follows:

in the domain

$$M_{ij} = -D_{ijkl}w_{,kl} \tag{1}$$

on the boundary

$$M_n = M_{ij}n_in_j \tag{2}$$

$$M_{nt} = M_{ij}n_it_j \tag{3}$$

here $n_i = \cos(n, x_i)$, $t_i = \cos(t, x_i)$ and the rigidity tensor

$$D_{ijkl} = D[\nu\delta_{ij}\delta_{kl} + (1-\nu)\,\delta_{ik}\delta_{jl}] \tag{4}$$

with δ_{ij} being the Kronecker symbol.

The shear forces can be expressed as:

in the domain

$$Q_i = M_{ij,j} \tag{5}$$

on the boundary

$$Q_n = Q_in_i = M_{ij,j}n_i \tag{6}$$

and the Kirchhoff equivalent shear force is

$$K_n = Q_n + M_{nt,s} \tag{7}$$

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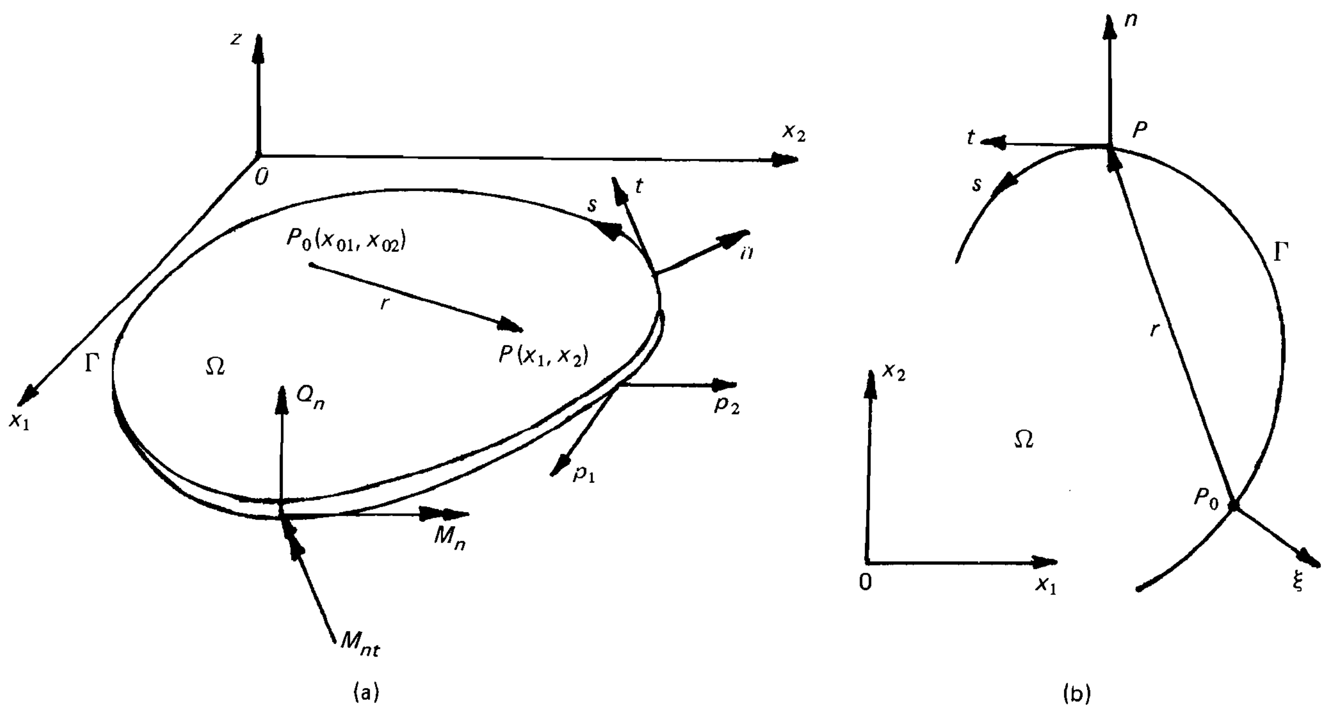

Figure 1. Notations

The fundamental differential equation for the elastic stability analysis of thin plate can be written as:

$$D\nabla^4 w = \lambda h \sigma_{ij} w_{,ij} \quad \forall p \in \Omega \tag{8}$$

in which $\nabla^4 = (\,)_{,kkll}$ is the biharmonic operator in $0x_1x_2$ co-ordinate system; λ the load factor and σ_{ij} the membrane stresses corresponding to external in-plane forces.

The boundary conditions of the plate stability analysis are:

for clamped boundary Γ^c

$$w = 0 \quad w_{,n} = 0 \tag{9}$$

for simply supported boundary Γ^s

$$w = 0 \quad M_n = 0 \tag{10}$$

for free boundary Γ^f

$$M_n = 0 \quad K_n = 0 \tag{11}$$

where

$$\Gamma^c \cup \Gamma^s \cup \Gamma^f = \Gamma$$

INTEGRAL EQUATION FORMULATION

The fundamental solution of biharmonic equation is:

$$w^* = w^*(P, P_0) = \frac{1}{8\pi} r^2 \ln r \tag{12}$$

where $r = |\overrightarrow{P_0P}|$ (Fig. 1a). w^* satisfies the following equation

$$\nabla^4 w^* = \delta(P, P_0) \quad \forall p \in \mathbb{R}^2 \tag{13}$$

here $\delta(P, P_0)$ is the Dirac δ-function which represents a unit force acting at point P_0 in an infinite plate.

Taking equations (8) and (13) into the following Rayleigh-Green identity:[6]

$$\int_\Omega (w\nabla^4 w^* - w^*\nabla^4 w)\, d\Omega = \frac{1}{D}\int_\Gamma [w^*K_n - wK_n^* + w_{,n}M_n^* - w_{,n}^*M_n]\, d\Gamma \tag{14}$$

where the boundary Γ is supposed to be smooth enough in the sense of Lyapunov, one can obtain

$$Dw(P_0) = \lambda h \int_\Omega \sigma_{ij} w_{,ij} w^*\, d\Omega + \int_\Gamma [w^*K_n - wK_n^* + w_{,n}M_n^* - w_{,n}^*M_n]\, d\Gamma \quad \forall P_0 \in \Omega \tag{15}$$

M_n^* and K_n^* are bending moment and Kirchhoff equivalent shear force corresponding to the fundamental solution w^*.

The integral equation (15) involves $w_{,ij} = \partial^2 w/\partial x_i\, \partial x_j$ in the domain. To reduce the order of differentiation of $w_{,ij}$, integrating the domain integral in (15) by parts and noticing that $\sigma_{ij,j} = 0$ in Ω, one can derive:

$$\int_\Omega \sigma_{ij} w_{,ij} w^*\, d\Omega = \int_\Omega \sigma_{ij} w_{,ij}^* w\, d\Omega + \int_\Gamma p_k(w_{,k}w^* - w_{,k}^*w)\, d\Gamma \tag{16}$$

in which $p_k = \sigma_{kl}n_l$ are tractions on the boundary. Substituting this expression into (15) and taking point P_0 toward boundary Γ give the following kind of integral equations:

$$\lambda h \int_\Omega \sigma_{ij} w^*_{,ij} w \,d\Omega + \lambda h \int_\Gamma p_k (w_{,k} w^* - w^*_{,k} w) \,d\Gamma$$

$$+ \int_\Gamma [w^* K_n - w K^*_n + w_{,n} M^*_n - w^*_{,n} M_n] \,d\Gamma$$

$$= \begin{cases} D w(P_0) & \forall P_0 \in \Omega \quad (17) \\ \tfrac{1}{2} D w(P_0) & \forall P_0 \in \Gamma \quad (18) \end{cases}$$

Let ξ denote the outward normal of boundary Γ at point P_0 (Fig. 1b). Differentiating both sides of equation (18) with respect to ξ, one obtains a supplementary integral equation as follows:

$$\lambda h \int_\Omega \sigma_{ij} w^*_{,\xi ij} w \,d\Omega + \lambda h \int_\Gamma p_k (w_{,k} w^*_{,\xi} - w^*_{,\xi k} w) \,d\Gamma$$

$$+ \int_\Gamma [w^*_{,\xi} K_n - w K^*_{n,\xi} + w_{,n} M^*_{n,\xi} - w^*_{,n\xi} M_n] \,d\Gamma$$

$$= \tfrac{1}{2} D w_{,\xi}(P_0) \qquad \forall P_0 \in \Gamma \tag{19}$$

Thus, the integral equations (17), (18) and (19) for the elastic stability analysis of thin plate have been established. This integral equation formulation is not related to the internal quantities $w_{,ij}$ $(i,j = 1,2)$ introduced in the original differential equation (8), and therefore eliminates three integral equation expressions for $w_{,ij}$ in the domain. Equations (17), (18) and (19) are similar to the integral equations in linear plate bending analysis by BEM and can be applied to solve elastic stability problems of plates with arbitrary plan forms and under arbitrary boundary conditions and in-plane load conditions.

BOUNDARY ELEMENT DISCRETISATION SCHEME

A boundary element discretisation scheme with constant elements will be formulated on the bases of integral equations (17), (18) and (19). This solution scheme can be further simplified for plates with clamped and simply supported edges or their combination.

The term $p_k w_{,k}$ in equations (17), (18) and (19) can be expressed as:

$$p_k w_{,k} = p_n w_{,n} + p_t w_{,t} \tag{20}$$

where p_n and p_t are components of traction in the direction of normal n and tangent t of boundary Γ respectively. For clamped and simply supported boundary conditions (9) and (10), $w_{,t} = 0$ exactly; but for free boundary condition (11), $w_{,t}$ is not equal to zero everywhere in general case. For the purpose of avoiding the introduction of new unknown variable, it is necessary to replace $w_{,t}$ with w by means of interpolation functions in the BEM formulation.

Suppose the boundary Γ is discretised into N constant boundary elements; the domain Ω into L cells (meshes), Fig. 2. For an element, the boundary variables w, $w_{,n}$, M_n and K_n are assumed to be constant and equal to their values at the mid-point of the element. Therefore $w_{,t} = 0$ approximately for free edges in this boundary element discretisation. On each cell the deflection w is also assumed to be constant, equal to its value at the centre of the cell.

By virtue of the discretisation scheme mentioned above, the two integral equations (18) and (19) will be reduced to the following algebraic equations:

$$[\mathbf{H}_1 \mathbf{H}_2] \begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\theta} \end{Bmatrix} = [\mathbf{G}_1 \mathbf{G}_2] \begin{Bmatrix} \mathbf{k} \\ \mathbf{m} \end{Bmatrix} + \lambda [\mathbf{F}_1 \mathbf{F}_2] \begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\theta} \end{Bmatrix} + \lambda \mathbf{C}\mathbf{y} \tag{21}$$

where w, $\boldsymbol{\theta}$, $\mathbf{m}$, $\mathbf{k}\,(\in \mathbb{R}^N)$ are vectors composed of w, $w_{,n}$, M_n and K_n on the boundary Γ respectively; $\mathbf{y}\,(\in \mathbb{R}^L)$ is the vector of w at centres of the internal cells; $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{G}_1$, $\mathbf{G}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$ $(\in \mathbb{R}^{2N \times N})$ and $\mathbf{C}\,(\in \mathbb{R}^{2N \times L})$ are relative coefficient matrices originated from the integrals of known functions on the boundary elements or internal cells. The integral equation (17) will be transformed into the following:

$$\mathbf{y} + [\mathbf{H}_{\Omega 1} \mathbf{H}_{\Omega 2}] \begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\theta} \end{Bmatrix} = [\mathbf{G}_{\Omega 1} \mathbf{G}_{\Omega 2}] \begin{Bmatrix} \mathbf{k} \\ \mathbf{m} \end{Bmatrix} + \lambda [\mathbf{F}_{\Omega 1} \mathbf{F}_{\Omega 2}] \begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\theta} \end{Bmatrix} + \lambda \mathbf{C}_\Omega \mathbf{y} \tag{22}$$

where $\mathbf{H}_{\Omega 1}$, $\mathbf{H}_{\Omega 2}$, $\mathbf{G}_{\Omega 1}$, $\mathbf{G}_{\Omega 2}$, $\mathbf{F}_{\Omega 1}$, $\mathbf{F}_{\Omega 2}$ $(\in \mathbb{R}^{L \times N})$ and $\mathbf{C}_\Omega\,(\in \mathbb{R}^{L \times L})$ are coefficient matrices.

Substitution of the boundary conditions (9), (10) and (11) into (21) and (22) produces:

$$\mathbf{G}\mathbf{x} = \lambda \mathbf{F}\mathbf{x} + \lambda \mathbf{C}\mathbf{y} \tag{23}$$

$$\mathbf{y} + \mathbf{G}_\Omega \mathbf{x} = \lambda \mathbf{F}_\Omega \mathbf{x} + \lambda \mathbf{C}_\Omega \mathbf{y} \tag{24}$$

where $\mathbf{x}\,(\in \mathbb{R}^{2N})$ is the vector of unknown boundary variables with unknown w or K_n in the first N components; matrices $\mathbf{G} \in \mathbb{R}^{2N \times 2N}$ and $\mathbf{G}_\Omega \in \mathbb{R}^{L \times 2N}$. Matrices $\mathbf{F}$ $(\in \mathbb{R}^{2N \times 2N})$ and $\mathbf{F}_\Omega$ $(\in \mathbb{R}^{L \times 2N})$ take different forms under different boundary conditions; for examples:

for clamped boundary condition

$$\mathbf{F} = \mathbf{0} \qquad \mathbf{F}_\Omega = \mathbf{0} \tag{25}$$

for simply supported boundary condition

$$\mathbf{F} = [\mathbf{0}\,\mathbf{F}_2] \qquad \mathbf{F}_\Omega = [\mathbf{0}\,\mathbf{F}_{\Omega 2}] \tag{26}$$

for free boundary condition

$$\mathbf{F} = [\mathbf{F}_1 \mathbf{F}_2] \qquad \mathbf{F}_\Omega = [\mathbf{F}_{\Omega 1} \mathbf{F}_{\Omega 2}] \tag{27}$$

where $\mathbf{0}$ are null matrices.

Equations (23) and (24) can be combined to form the following eigenvalue problem:

$$\begin{bmatrix} \mathbf{F} & \mathbf{C} \\ \mathbf{F}_\Omega & \mathbf{C}_\Omega \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{y} \end{Bmatrix} = \mu \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{G}_\Omega & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{y} \end{Bmatrix} \tag{28}$$

where $\mu = 1/\lambda$ and $\mathbf{I}\,(\in \mathbb{R}^{L \times L})$ is a unit matrix. This equation is a generalised eigenvalue problem which can be converted to:

$$\begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{G}_\Omega & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F} & \mathbf{C} \\ \mathbf{F}_\Omega & \mathbf{C}_\Omega \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{y} \end{Bmatrix} = \mu \begin{Bmatrix} \mathbf{x} \\ \mathbf{y} \end{Bmatrix} \tag{29}$$

The dimension of this standard eigenvalue problem is $(2N + L)$.

Substituting equation (23) into (24) one can obtain:

$$(\mathbf{C}_\Omega - \mathbf{G}_\Omega \mathbf{G}^{-1} \mathbf{C})\,\mathbf{y} + (\mathbf{F}_\Omega - \mathbf{G}_\Omega \mathbf{G}^{-1} \mathbf{F})\,\mathbf{x} = \mu \mathbf{y} \tag{30}$$

For the plate clamped on edges, $\mathbf{F} = \mathbf{0}$ and $\mathbf{F}_\Omega = \mathbf{0}$, the above equation reduces to:

$$(\mathbf{C}_\Omega - \mathbf{G}_\Omega \mathbf{G}^{-1} \mathbf{C})\,\mathbf{y} = \mu \mathbf{y} \tag{31}$$

The dimension of this eigenvalue problem is L, much lower than that of problem (29).

For the plate with simply supported edges, $\mathbf{F}$ and $\mathbf{F}_\Omega$ are not null matrices, but in the numerical computation of all the examples with simply supported edges, studied in this paper, the matrix $(\mathbf{F}_{\Omega} - \mathbf{G}_{\Omega}\mathbf{G}^{-1}\mathbf{F})$ is a null matrix within the range of effective numbers in all cases. Therefore, for plates with simply supported edges, eigenvalue problem (31) can be solved instead of (29), which will save much effort of computation. This is an interesting phenomenon and needs to be further investigated so as to understand the physical and mathematical meanings behind it.

Thus, the boundary element discretisation scheme with constant elements has been established based on the integral equations (17), (18) and (19). For plates under general boundary conditions, formulation (29) can be utilised; for plates under clamped and simply supported boundary conditions or their combination, the more efficient formulation (31) is preferred. In the studies of numerical examples presented in this paper, the critical force factors λ_{cr} are determined by the highest eigenvalues μ_{max} of eigenvalue problem (31). For the plate with simply supported edges, problem (29) is also solved and the results show that its L eigenvalues are the same as those of problem (31) and another $2N$ eigenvalues are zeros.

NUMERICAL RESULTS AND DISCUSSIONS

Numerical examples of square and circular plates under various boundary and in-plane load conditions were studied with constant elements to verify the BEM formulation (29) and (31) described above. The boundary element division and the interior meshes are portrayed in Fig. 2. All the numerical computations were performed on the microcomputer IBM PC.

Numerical results for square and circular plates are presented in Table 1. For square plate with side length a, the critical load is expressed by $P_{cr} = \lambda_{cr}\pi^2 D/a^2$ and the number of internal points (i.e. the number of internal cells) $L = (N/4)^2$. For circular plate with radius R, $P_{cr} = \lambda_{cr} D/R^2$ and $L = 37, 81, 141, 217$ when $N = 16, 24, 32, 40$ respectively. The critical load factors computed in this paper are compared with the analytical solutions and the BEM results presented in ref. 28, and good agreement is achieved. It

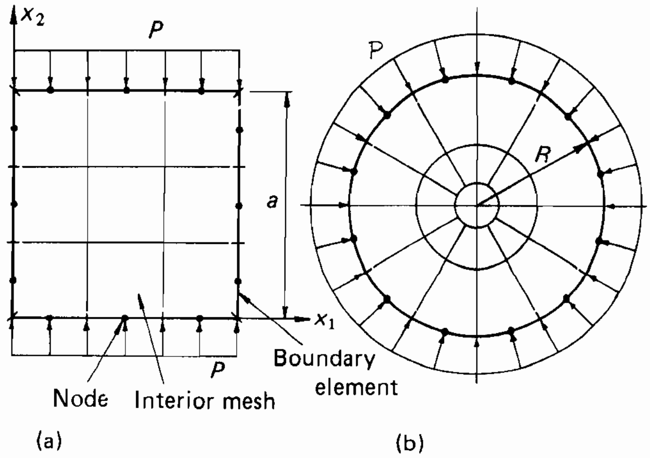

Figure 2. Boundary element division and interior meshes; (a) square plate, (b) circular plate

should be noted that the dimension of the matrix in the final algebraic eigenvalue problem is $3L$ for the BEM formulation in ref. 28, but is L for the BEM formulation in this paper.

The numerical results demonstrate that the BEM formulation developed in this paper is correct and efficient. Acceptable data, in view of engineering applications, can be obtained with few constant elements on a microcomputer. To further improve the accuracy of results, boundary element and interior mesh discretisations with higher order can be introduced, and problems such as the twist moment discontinuity at corners, which has been neglected in this paper for its minor effect on solutions away from the corners, can be taken into account. The distribution of in-plane stresses, which has been input with the exact solutions of the simple examples, can be first determined in general cases by using the BEM solution program for two-dimensional elasticity problems.

As it was expected, the matrices in eigenvalue problems (29) and (31) are found to be positively definite matrices

Table 1. Critical force factors of square and circular plates under various load and boundary conditions ($\nu = 0.3$)

Plate shapes	Load conditions	Boundary conditions	BEM solutions of this paper $N = 16$	$N = 24$	$N = 32$	$N = 40$	Ref. 28 ($N = 36$, $L = 64$)	Exact values[29]
Square plates	$p_n(x) = -P$ on edges $y = 0, a$	SS on all edges	4.055	4.039	4.026	4.018	4.13	4.00
		C on all edges	10.910	10.622	10.401	10.286	10.51	10.07
		C on $x = 0, a$ SS on $y = 0, a$	8.181	8.024	7.900	7.831	8.01	7.80
	$p_n = -P$ on all edges	SS on all edges	2.028	2.019	2.013	2.009	2.05	2.00
		C on all edges	5.925	5.609	5.479	5.416	5.43	5.33
	$p_n(x) = -Px/a$ on edges $y = 0, a$	SS on all edges	7.989	7.910	7.871	7.851	8.00	7.80
	$p_n(x) = -P(1 - 2x/a)$ on edges $y = 0, a$	SS on all edges	24.643	25.949	25.832	25.727	26.61	25.60
	$p_t(x) = -P$ on edges $y = 0, a$; $p_t(y) = P$ on edges $x = 0, a$	SS on all edges	10.600	9.524	9.371	9.338	9.67	9.34
		C on all edges	17.994	15.383	14.885	14.742	14.90	14.71
		C on $x = 0, a$ SS on $y = 0, a$	14.986	13.096	12.748	12.647	12.53	12.28
Circular plates	$p_n = -P$	SS on edge	4.745	4.494	4.394	4.342	–	4.20
		C on edge	15.147	14.958	14.852	14.798	–	14.68

by the numerical computation (all eigenvalues are real positive numbers). Therefore simple methods for eigenvalue calculation, such as the vector iteration method, can be applied, which will take much less CPU time than the HQR method employed in studies of the above examples.

CONCLUSIONS

It can be concluded that the integral equation formulation developed in this paper is effective and efficient for the elastic stability analysis of thin plate with arbitrary plan form and under arbitrary load and boundary conditions. The boundary element discretisation scheme based on this formulation with constant elements is shown to be applicable and especially efficient for plates with clamped and simply supported edges or their combination.

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