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Computational Materials Science 21 (2001) 86–94

COMPUTATIONAL MATERIALS SCIENCE

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Multiple-cell modeling of fiber-reinforced composites with the presence of interphases using the boundary element method

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Received 20 July 2000; received in revised form 24 October 2000; accepted 1 November 2000

Abstract

In this paper, an advanced boundary element method (BEM) with thin-body capabilities is applied to model multiple cells of fiber-reinforced composites with the consideration of the interphases. Effects of the multiple-cell models, as compared to the unit-cell model, in determining the effective material constants in the transverse plane, are studied. In this BEM approach, the interphases are modeled as thin elastic layers based on the elasticity theory, as opposed to spring-like models in the previous BEM and some models based on the finite element method (FEM). The BEM approach to the multiple-cell modeling is compared with the FEM approach. The advantages and disadvantages of the BEM as compared with the FEM for the analysis of fiber-reinforced composites are discussed. It is shown that the developed BEM is very accurate and efficient in the modeling and analysis of fiber-reinforced composites, and that different cell models can have marked influences on the evaluations of the effective modulus of fiber-reinforced composites.

Keywords: Fiber-reinforced composites; Interphases; Multiple-cell models; Boundary elements

1. Introduction

Study of the interphases, which are thin layers of a third material between the fiber and matrix materials (Fig. 1), is very important because they play a crucial role in the functionality and reliability of the composite materials [1,2]. Effective utilization of the strength and stiffness of the fiber-reinforced composites depends on efficient load transfers from the matrix to fibers through these interphases. It is therefore essential to understand the effects of the interphases in the fiber-reinforced composites, in order to provide some guidelines in improving the design of the composite materials. Many investigators have studied the influences of the interphase thickness and material properties on the effective Young's moduli of fiber-reinforced composites using analytical, experimental or computational approaches, as shown, for example, in [1–10] and the references therein.

Computational methods using the finite element method (FEM) or the boundary element method (BEM) are effective ways to study the micromechanical behaviors of composite materials (see, e.g. [3–5,9–15]). The boundary integral equation/boundary element method (BIE/BEM), pioneered in [16] for elasticity problems, has been demonstrated to be a viable alternative to the FEM for many problems in engineering, due to its features

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0927-0256/01/$ - see front matter
PII: S0927-0256(00)00218-4

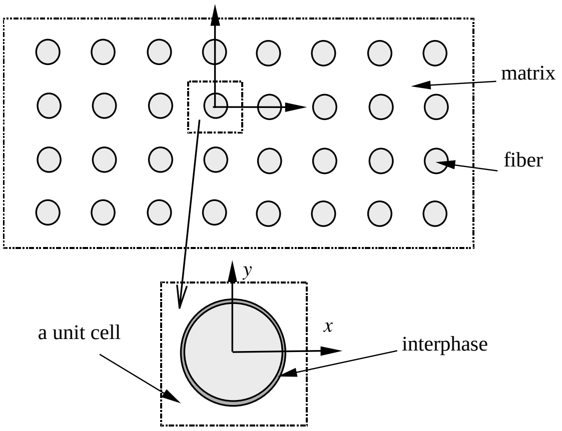


Fig. 1. Interphases in a fiber-reinforced composite.

of boundary-only discretization and high accuracy (see, e.g. [17–20]). The high accuracy and efficiency of the BEM for stress analysis, especially in fracture mechanics [18,21], is well recognized because of its semi-analytical nature and boundary-only discretization. The discretization errors in the BEM are mainly confined to the boundary of the material domain and interfaces between different materials. The meshing for the BEM is also much more efficient than that for other domain-based methods, as will be demonstrated in this paper.

Recently, an advanced BEM with thin-body capabilities was developed for the studies of the interphases in fiber-reinforced materials [22,23], and thin films or coatings [24,25]. In this BEM approach, the interphases are modeled as thin elastic layers using the elasticity theory, as opposed to the spring-like models in the previous BEM [3,4,15] and some FEM work. The developed BEM approach is found to be extremely accurate and efficient for the analysis of thin and layered structures. By employing much fewer boundary elements (less than 200) for a whole unit-cell model (Fig. 2), the developed BEM can provide accurate stress results for which the FEM has to employ more than 3500 elements for only a quarter model of the same unit-cell as reported in [10]. However, in the work reported in [22], only unit-cell models of the fiber-reinforced composites, containing only one fiber with the surrounding matrix and interphase, are considered. Interactions among the fibers are not taken into account and thus the effects of these interactions in determining the effective material constants cannot be studied by using these unit-cell models.

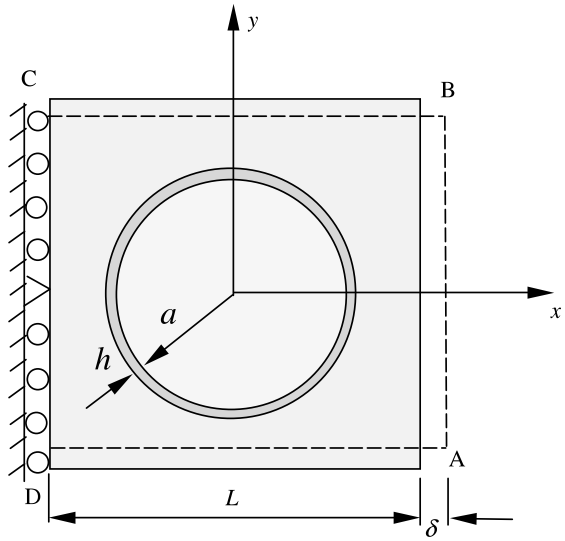


Fig. 2. A square unit-cell model under tension.

In this paper, this advanced BEM approach [22] is extended to model multiple cells (fibers) of fiber-reinforced composites with the consideration of the interphases. Effects of the multiple-cell models, as compared to the unit-cell model, in determining the effective material constants in the transverse plane, are studied. The BEM approach to the multiple-cell modeling is compared with the FEM approach. The advantages and disadvantages of the BEM as compared with the FEM for the analysis of fiber-reinforced composites are discussed. It is shown that the developed BEM is very accurate and efficient in the modeling and analysis of the fiber-reinforced composites, and that different cell models can have marked influences on the evaluations of the effective modulus of fiber-reinforced composites.

2. The boundary integral equation formulation

The following conventional boundary integral equation for isotropic, linearly elastic materials [16] is applied in this study (index notation is used here):

$$C_{ij}(P_0)u_j^{(\beta)}(P_0) = \int_S [U_{ij}^{(\beta)}(P,P_0)t_j^{(\beta)}(P) - T_{ij}^{(\beta)}(P,P_0)u_j^{(\beta)}(P)]\,\mathrm{d}S(P), \tag{1}$$

in which $u_i^{(\beta)}$ and $t_i^{(\beta)}$ are the displacement and traction fields, respectively; $U_{ij}^{(\beta)}(P,P_0)$ and $T_{ij}^{(\beta)}(P,P_0)$ the displacement and traction kernels (Kelvin's solution or the fundamental solution), respectively; P the field point and P_0 the source point; and S the boundary of a single material domain V. $C_{ij}(P_0)$ is a constant coefficient matrix depending on the smoothness of the curve S at the source point P_0. The superscript β on the variables in Eq. (1) signifies the dependence of these variables on the material domain. The expressions for the kernel functions $U_{ij}^{(\beta)}(P,P_0)$ and $T_{ij}^{(\beta)}(P,P_0)$, which contain the material constants, can be found in [22] or any other references on the BEM (see, e.g. [17,19,20]).

BIE (1) is applied to each material domain (matrix, fiber and interphase), which relates the boundary displacement and traction fields in that domain only. The resulting BIEs from each domain are coupled through the interface conditions [22]. The continuity of the displacement and traction is imposed at the perfectly bonded interface. The discretization procedures to arrive at the linear systems of equations based on BIE (1) can be found in any references on the BEM (see, e.g. [17,19,20]). The final system of linear algebra equations based on BIE (1) for the composite material problems considered here can be written as [22]:

$$\begin{bmatrix} \mathbf{T}_1^{(\mathrm{f})} & -\mathbf{U}_1^{(\mathrm{f})} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{T}_1^{(\mathrm{i})} & \mathbf{U}_1^{(\mathrm{i})} & \mathbf{T}_2^{(\mathrm{i})} & -\mathbf{U}_2^{(\mathrm{i})} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_2^{(\mathrm{m})} & \mathbf{U}_2^{(\mathrm{m})} & \mathbf{T}_3^{(\mathrm{m})} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_1 \\ \mathbf{t}_1 \\ \mathbf{u}_2 \\ \mathbf{t}_2 \\ \mathbf{u}_3 \end{Bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{U}_3^{(\mathrm{m})} \end{bmatrix} \{\mathbf{t}_3\}, \tag{2}$$

in which $\mathbf{U}$ and $\mathbf{T}$ are matrices generated from the $U_{ij}^{(\beta)}(P,P_0)$ and $T_{ij}^{(\beta)}(P,P_0)$ kernels, respectively; $\mathbf{u}$ and $\mathbf{t}$, the nodal displacement and traction vectors, respectively, at the interfaces or outer boundary. The superscripts indicate the material domain (f = fiber, i = interphase, and m = matrix), while the subscripts indicate the interface or boundary (1 = fiber–interphase interface, 2 = interphase–matrix interface, and 3 = matrix boundary) on which the integration is performed. Details of coupling the BIEs from each domain and the numerical implementations can be found in [22,24].

In the recent work in [22], interphases in unidirectional fiber-reinforced composites under transverse loading are modeled successfully by the BEM based on the elasticity theory. The interphases are regarded as elastic layers between the fiber and matrix, as opposed to the spring-like models in the BEM literature. Both cylinder and square unit-cell models of the fiber–interphase–matrix systems are considered. The effects of varying the modulus and thickness (including nonuniform thickness) of the interphases with different fiber volume fractions are investigated. Numerical results demonstrate that the developed BEM is very accurate and efficient in determining the interface stresses and effective elastic moduli of fiber-reinforced composites with the presence of interphases of arbitrarily small thickness and nonuniform thickness. Interface cracks at the interphase regions are also considered using this BEM approach [23]. However, in the work reported in [22], only unit-cell models of the fiber-reinforced composites, containing only one fiber and the surrounding matrix and interphase, is considered. Interactions among the fibers are not taken into account and thus the effects of these interactions in determining the effective materials constants cannot be studied, using these unit-cell models.

In this paper, the BIE as given in Eq. (1) and with the thin-body capabilities developed in [22,24] for 2-D thin elastic materials is employed to study the multiple-cell models of fiber-reinforced composites with the presence of the interphases. The isoparametric quadratic boundary (line) elements are applied in this study. The finite element approach is also tested in this study for this type of analysis of composite materials using the commercial FEM software package *ANSYS*. Comparison of the BEM approach with that of the

FEM is investigated carefully regarding the modeling efficiency and solution accuracy.

3. Numerical examples

The multiple-cell models used in this study are the 2×2 and 3×3 models as shown in Fig. 3. Their corresponding BEM and FEM discretizations are shown in Figs. 4 and 5. More fibers can be included in the model, e.g., using an $n \times n$ model with $n > 3$. The fibers can also be arranged in a different pattern, e.g., in hexagon or randomly. Studies of these cases are readily achievable with the developed BEM and will be left to future investigations.

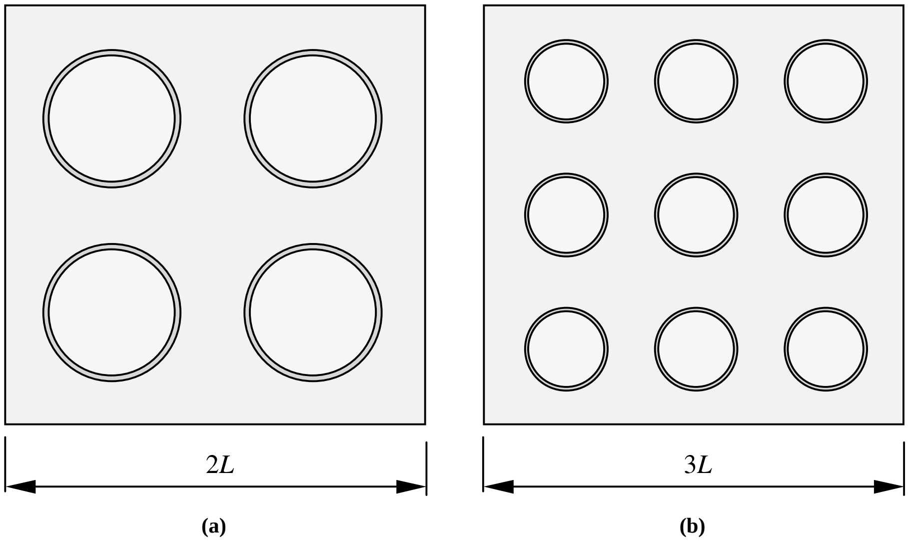


Fig. 3. Square multiple-cell models: (a) 2×2 model; (b) 3×3 model.

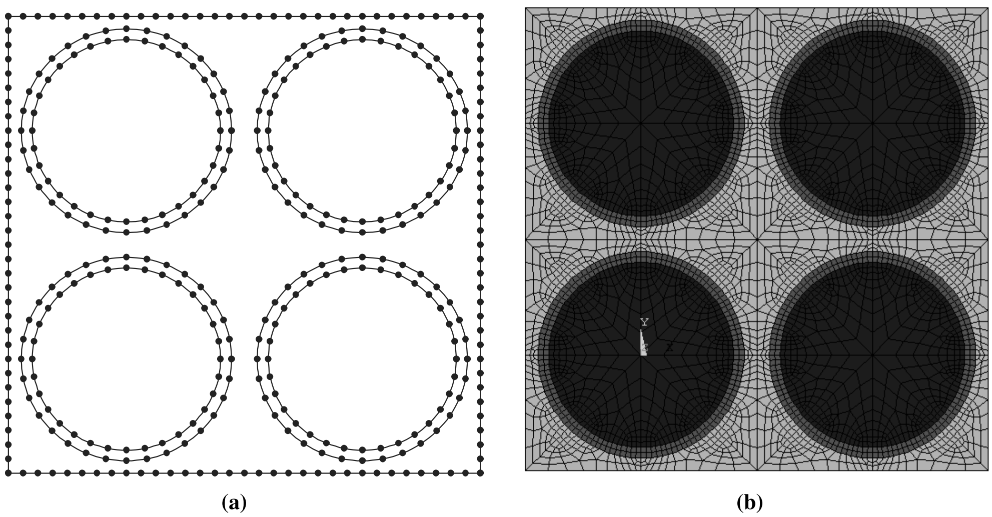
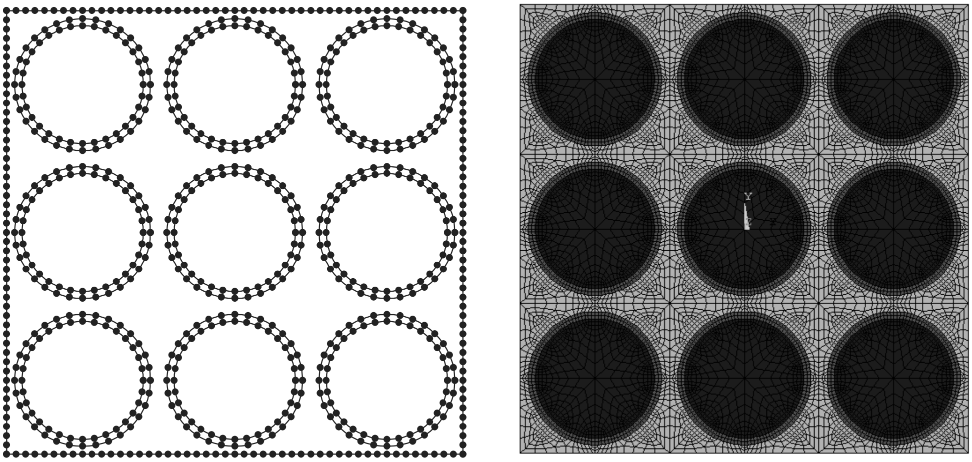


Fig. 4. Discretizations of the 2×2 model: (a) BEM mesh; (b) FEM mesh.



(a) (b)

Fig. 5. Discretizations of the 3×3 model: (a) BEM mesh; (b) FEM mesh.

3.1. Effect of the multiple-cell models on the effective Young's modulus

Effects of the multiple-cell models in determining the effective Young's modulus with varying interphase property as compared to the unit-cell model are studied first.

The square unit-cell model and multiple-cell (2×2 and 3×3 cell) models under tension in the x-direction are shown in Figs. 2 and 3, respectively. The properties of the constituent materials (epoxy–matrix, e-glass fiber) considered are:

for fiber $E^{(f)} = 84.0$ GPa, $\nu^{(f)} = 0.22$;

for interphase $E^{(i)} = 4.0 \sim 12.0$ GPa, $\nu^{(i)} = 0.34$;

for matrix $E^{(m)} = 4.0$ GPa, $\nu^{(m)} = 0.34$;

and $a = 8.5$ μm, $b = a + h, L = 21.31$ μm, fiber volume fraction $V_f = 50\%$ (Fig. 2). Young's modulus for the interphase is changing in the range between 4.0 and 12.0 GPa. In the unit-cell BEM model, a total of 64 quadratic line elements are used with 16 elements on each of the two circular interfaces and 32 elements on the outer boundary in the BEM model as compared with the FEM model with 1,128 quadratic 2-D elements (an FEM mesh with converged results). In the 2×2 multiple-cell model, a total of 192 quadratic elements are used with 16 elements on each of the eight circular interfaces and 64 elements on the outer boundary in the BEM model (Fig. 4(a)) as compared with the FEM model (Fig. 4(b)) with 4,512 quadratic elements. In the 3×3 multiple-cell model, a total of 384 quadratic elements are used with 16 elements on each of the eighteen circular interfaces and 96 elements on the outer boundary in the BEM model (Fig. 5(a)) as compared with the FEM model (Fig. 5(b)) with 10,152 quadratic elements.

Table 1 shows the effective Young's modulus obtained by using the unit-cell and multiple-cell BEM models, as compared with the corresponding FEM models for the thickness $h = 1.0$ μm (See [22] for the expressions for the effective transverse Young's modulus of fiber-reinforced composites under the plane-stain condition). The BEM results are very close to the corresponding FEM results with all the differences within 1%.

Finite element contour plot for stress σ_x is presented in Fig. 6. We observe that the straight-line condition along the cell edges is not satisfied by the free-traction assumption along the top and bottom edges of the model. However, it is shown in [22] that the different boundary conditions along the top and bottom edges of the square model (free-traction or straight-line conditions) have negligible influences on the calculation of the effective Young's modulus.

Table 1
BEM and FEM results of the effective Young's modulus E_x (GPa) for different interphase Young's modulus

Model		$E^{(i)} = 4.0$ GPa	$E^{(i)} = 6.0$ GPa	$E^{(i)} = 8.0$ GPa	$E^{(i)} = 12.0$ GPa
(1) Unit-cell model	BEM	11.57	13.05	13.98	15.09
	FEM	11.59	13.04	13.94	15.01
(2) 2×2 model	BEM	11.63	13.07	13.96	15.01
	FEM	11.62	13.06	13.97	15.04
(3) 3×3 model	BEM	12.30	13.84	14.79	15.92
	FEM	12.33	13.86	14.81	15.92

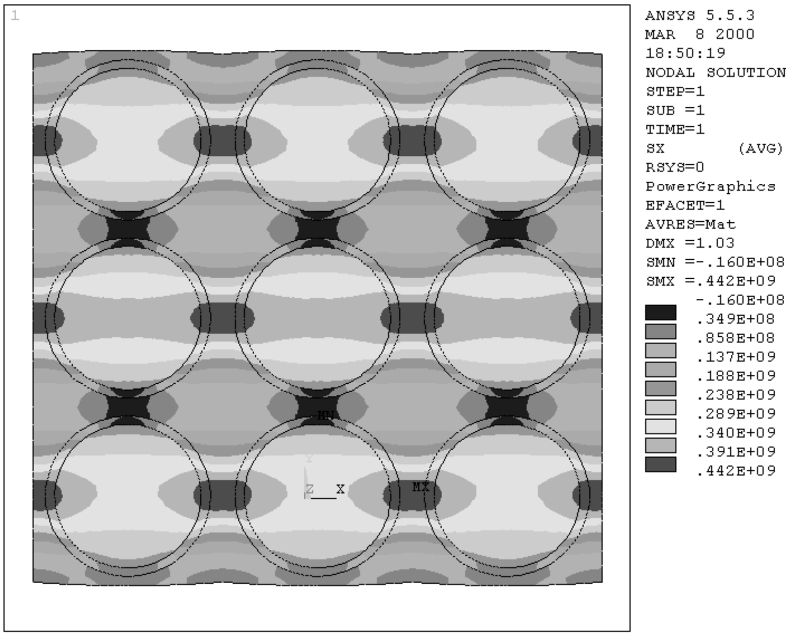


Fig. 6. FEM (*ANSYS*) contour plot for stress σ_x.

It is shown in the Table 1 that the material properties of the interphase and modeling using the multiple-cell models do have influences on the effective Young's modulus of the composites. The Young's modulus results of the unit-cell model differ slightly with those of the 2×2 multiple-cell model (within 1%). However, the differences between the results of the 3×3 multiple-cell model and those of the unit-cell model increase to around 6%. With the increase of numbers of fibers involved in the model, the difference may further increase. We can observe the results more clearly from Fig. 7, which shows the influence of multiple-cell model on the effective Young's modulus. From Fig. 7, we also notice that the effect of

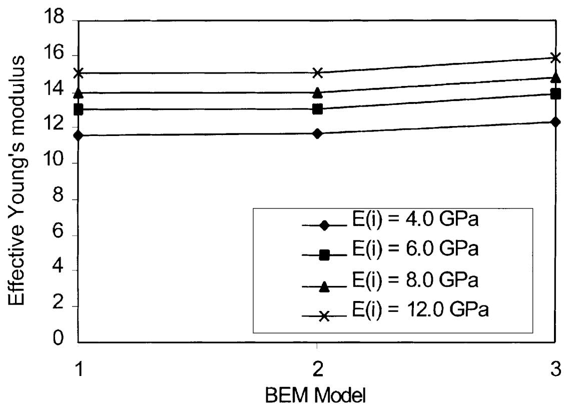


Fig. 7. Influence of multiple-cell model on the effective Young's modulus.

Table 2
BEM results of the effective Young's modulus E_x (GPa) for different interphase thickness h

BEM Model	$h = 1.0$ μm	$h = 0.7$ μm	$h = 0.5$ μm	$h = 0.2$ μm
(1) Unit-cell model	13.05	12.58	12.28	11.85
(2) 2 × 2 model	13.07	12.82	12.52	12.09
(3) 3 × 3 model	13.84	13.35	13.04	12.59

multiple-cell models on the effective transverse Young's modulus will be nearly the same for different interphase Young's modulus.

3.2. Effect of the interphase thickness

In the above results, the only thickness considered is $h = 1.0$ μm, which is relatively large compared with the fiber radius ($a = 8.5$ μm). If a smaller thickness were used in the FEM model, a much larger number of elements would have been needed in order to avoid large aspect ratios in the FEM mesh. However, for the boundary element method employed here, the same number of elements can be used, no matter how small the thickness of the interphase is. Table 2 shows the effective Young's modulus for unit-cell model and multiple-cell models with the change of the interphase thickness from 1.0 to 0.2 μm when $E^{(i)} =$ 6.0 GPa. The influence of thickness on the effective Young's modulus is shown in Fig. 8, from which we observe that the effective Young's modulus decreases with the decrease of interphase thickness. The influence of multiple-cell models on the effective Young's modulus is nearly the same for different interphase thickness.

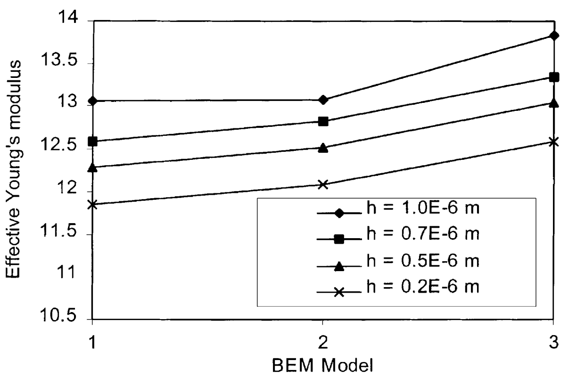


Fig. 8. Influence of thickness on the effective Young's modulus.

4. Discussions

Numerical studies in this paper show that the thickness, material properties of the interphases, and the number of fibers involved in the model have marked influences on the analysis of the micromechanical behaviors of the composites. In these studies, boundary element method shows great potential in dealing with structures with thin-layers due to its boundary-based nature as compared with finite element method which is domain-based. When the thickness of the interphase is changed, the BEM mesh can be updated easily, while for the FEM totally different meshes need to be generated for different interphase thickness. When the interphase thickness is relatively small, an extremely large number of elements need to be used in the FEM model. Hence, using the FEM may not even be feasible to deal with such problems, if the computing resources are limited. With much fewer boundary (line) elements, the BEM distinguishes itself in the study of the materials or structures with thin shapes, regarding the modeling efficiency and solution accuracy.

However, at present, the developed BEM solver runs several times slower than the commercial FEM software *ANSYS* used in this study for the same problem. This is due to the fact that a lot of numerical integrations need to be done in the BEM approach in order to form the coefficient matrix and these integrations must be done accurately to ensure the accuracy of the BEM results. Optimization of the integration process and solution methods in the BEM is possible, such as using the new multipole expansion techniques and iterative solvers. These investigations are under way in order to improve the solution efficiency of the developed BEM. On the other hand, the commercial FEM software has been improved significantly over the time, and thus has been very much optimized regarding the solution efficiency. Even if the BEM after the optimization still runs slower than an FEM software, the efficiency of the BEM in the modeling stage (human time) can well offset the longer time in the solution process (computer time). The convenience of the BEM in handling the shell-like structures [26–28], such as the interphases in composite materials, and the accuracy of the BEM, make the BEM a very attractive numerical tool for the analysis of such materials or structures. With further improvements and the development of an easy-to-use graphical-user interface (GUI) for the developed BEM, it can become an efficient, accurate, and yet robust numerical analysis tool for the materials research and development.

5. Conclusion

An advanced boundary element method has been developed to study the interactions of multiple fibers in the composites with the presence of the interphases. Influences of the interphase thickness and material properties on the effective Young's modulus in the transverse plane have been investigated. The numerical results demonstrate that the developed BEM is very accurate and efficient for the analysis of multiple-cell models of fiber-reinforced composites, with the presence of the interphases. Extensions of the BEM to consider the thermal loading, multi-layered materials (coatings and thin films), various interface cracks and 3-D models will be interesting topics and can be carried out readily.

Acknowledgements

The support for this research by the National Science Foundation under the grant CMS9734949 is gratefully acknowledged. The authors thank the reviewers of the manuscript for their helpful comments.

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