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Analytical integration of the moments in the diagonal form fast multipole boundary element method for 3-D acoustic wave problems

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ABSTRACT

A diagonal form fast multipole boundary element method (BEM) is presented in this paper for solving 3-D acoustic wave problems based on the Burton–Miller boundary integral equation (BIE) formulation. Analytical expressions of the moments in the diagonal fast multipole BEM are derived for constant elements, which are shown to be more accurate, stable and efficient than those using direct numerical integration. Numerical examples show that using the analytical moments can reduce the CPU time by a lot as compared with that using the direct numerical integration. The percentage of CPU time reduction largely depends on the proportion of the time used for moments calculation to the overall solution time. Several examples are studied to investigate the effectiveness and efficiency of the developed diagonal fast multipole BEM as compared with earlier p^3 fast multipole method BEM, including a scattering problem of a dolphin modeled with 404,422 boundary elements and a radiation problem of a train wheel track modeled with 257,972 elements. These realistic, large-scale BEM models clearly demonstrate the effectiveness, efficiency and potential of the developed diagonal form fast multipole BEM for solving large-scale acoustic wave problems.

1. Introduction

The boundary element method (BEM) based on the boundary integral equation (BIE) formulation can be used to analyze acoustic wave problems effectively, such as in noise prediction for automobiles [1], high speed trains [2], airplanes [3], and underwater structures [4]. Several of the earlier work laid the foundation for applying the BIE/BEM to solve acoustic problems [5–10]. Especially, the work by Burton and Miller in Ref. [7] has been regarded as a classical one, which provides an elegant way to overcome the so called fictitious eigenfrequency difficulties existing in the conventional BIE for exterior acoustic wave problems [11,12].

In the last decade, the focus of the research has been on developing fast solution methods for efficiently solving large-scale BEM models for acoustic problems. The fast multipole method (FMM) is one of the most promising fast solution methods for the BEM. FMM was first pioneered by Rokhlin [13] and further developed by Greengard and Rokhlin [14] for fast simulation of large particle fields in physics. The FMM can improve the matrix-vector multiplication dramatically from $O(N^2)$ to $O(N)$ or $O(N \log N)$ with N being the number of degrees of freedom. Later on, a diagonal form FMM for Helmholtz problems was proposed by Rokhlin [15] as well. Since then, many research works have been published to improve and extend the applicability of the FMM for Helmholtz equations. Epton and Dembart [16] presented a concise summary of multipole translations for 3-D Helmholtz equations. Rahola [17] gave an error analysis of the FMM by considering both truncation error of the kernel expansion and the errors from the use of numerical integration in diagonal translation theorem. Darve [18] provided a rigorous mathematical approach on the estimation of the truncation error. Besides the above error considerations, Koc et al. [19] also analyzed the interpolation error in multilevel FMM. To accelerate the low frequency FMM, Greengard et al. [20] used the combination of evanescent and propagate mode to reduce the computation cost. Darve and Have [21] proposed a stable plane wave expansion, which uses the singular-value decomposition method to represent the evanescent kernel for the low frequency FMM. Gumerov and Duraiswami [22,23] extended the recurrence relations reported in Chew's paper [24] to develop a general recursive method for obtaining the translation matrices, the resulting approach is generally termed as p^3 FMM for solving the Helmholtz equation (with p being the order of the expansion). Adaptive algorithms for the FMM were also developed to speed up the solutions for 3-D full- and half-space acoustic problems [25–27]. The fast multipole BEM for solving structural-acoustic interaction problems was developed by

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Gaul and Fisher [28,29]. Hybrid FMMs were developed recently by Cheng et al. [30] and Gumerov and Duraiswami [31], which are stable for a wide range of frequencies. The former switches to different representations at low and high frequencies, while the latter is based on a rotation – coaxial translation – back rotation scheme. More information about the fast multipole BEM in general can be found in a review article [32], a tutorial [33], and the first textbook [34].

A new diagonal fast multipole BEM for solving 3-D Helmholtz equation with analytical integration of the moments is presented in this paper. The BEM is based on the Burton–Miller's BIE formulation [7], which has no fictitious eigenfrequency difficulties in solving exterior acoustic problems. The implementation of the diagonal FMM is also based on the adaptive fast multipole BEM given in Ref. [25] for 3-D full-space acoustic problems, in Ref. [26] for 3-D half-space acoustic problems, and in Ref. [27] for a new definition of the interaction list. The FMM used in Refs. [25–27] is valid for all frequencies, but is less efficient than the diagonal form FMM at high frequencies (e.g., with the nondimensional wavenumber ka above 300), since the translation complexity is at best $O(p^3)$ in Refs. [25–27] (with p being the expansion order at each tree level). The developed diagonal fast multipole BEM with the analytical integration of the moments is a significant improvement of the above mentioned fast multipole BEM, which can fill the gap in the analysis of high-frequency acoustic problems.

The rest of the paper is organized as follows. First the BIE formulation is reviewed in Section 2. The diagonal form FMM is presented in Section 3. Then, the fast multipole BEM algorithm is described in Section 4. The analytical moment formulation for the diagonal FMM is presented in Section 5. In Section 6, several numerical examples are given to demonstrate the capability of the proposed diagonal form FMM in modeling large-scale acoustic problems. Section 7 concludes this paper.

2. Boundary integral equations

The governing equation in the frequency domain of time-harmonic acoustic waves in a homogeneous isotropic acoustic medium E is described by the following Helmholtz equation:

$$\nabla^2\varphi(\mathbf{x})+k^2\varphi(\mathbf{x})=0, \quad \forall\mathbf{x}\in E, \tag{1}$$

where $\varphi(\mathbf{x})$ is the sound pressure at point $\mathbf{x}$, k is the wave number defined by $k=\omega/c$, with ω being the angular frequency and c the sound speed in medium E. Using Green's second identity, the solution of Eq. (1) can be expressed by an integral representation:

$$\varphi(\mathbf{x})=\int_S\left[G(\mathbf{x,y})q(\mathbf{y})-\frac{\partial G(\mathbf{x,y})}{\partial n(\mathbf{y})}\varphi(\mathbf{y})\right]dS(\mathbf{y})+\varphi^I(\mathbf{x}), \quad \forall\mathrm{x}\in E, \tag{2}$$

where $\mathbf{x}$ is the source point and $\mathbf{y}$ is the field point on boundary S, $q(\mathbf{y})$ is defined as $q(\mathbf{y})=\partial\varphi(\mathbf{y})/\partial n(\mathbf{y})$ where the unit normal vector $n(\mathbf{y})$ on boundary S is defined to point outwards from E. Incident wave $\varphi^I(\mathbf{x})$ will not be presented for radiation problems. In this paper, the time convention adopted is using the factor $e^{-i\omega t}$, correspondingly, the free-space Green's function G for 3-D problems is given by

$$G(\mathbf{x,y})=\frac{e^{ikr}}{4\pi r} \text{ with } r=|\mathbf{x}-\mathbf{y}|. \tag{3}$$

Letting point $\mathbf{x}$ approach the boundary leads to the following conventional boundary integral equation (CBIE):

$$c(\mathbf{x})\varphi(\mathbf{x})=\int_S\left[G(\mathbf{x,y})q(\mathbf{y})-\frac{\partial G(\mathbf{x,y})}{\partial n(\mathbf{y})}\varphi(\mathbf{y})\right]dS(\mathbf{y})+\varphi^I(\mathbf{x}), \quad \forall\mathbf{x}\in S, \tag{4}$$

where constant $c(\mathbf{x})=1/2$ if S is smooth around point $\mathbf{x}$. There is a defect with Eq. (4) concerning the non-uniqueness of the solution of an exterior acoustic problem at the eigenfrequency associated with the corresponding interior problem. To deal with the non-uniqueness difficulties, Burton and Miller [7] proposed a method by combining the CBIE and the normal derivative of the CBIE. Taking the derivative of integral representation Eq. (2) with respect to the normal at the field point $\mathbf{x}$ and also letting point $\mathbf{x}$ approach the boundary lead to the following hypersingular boundary integral equation (HBIE):

$$c(\mathbf{x})q(\mathbf{x})=\int_S\left[\frac{\partial G(x,y)}{\partial n(x)}q(\mathbf{y})-\frac{\partial^2 G(x,y)}{\partial n(y)\partial n(x)}\varphi(\mathbf{y})\right]dS(\mathbf{y})+q^I(\mathbf{x}), \quad \forall\mathbf{x}\in E, \tag{5}$$

where $q^I(\mathbf{x})=\partial\varphi^I(\mathbf{x})/\partial n(\mathbf{x})$. For an exterior problem, Eqs. (4) and (5) have a different set of fictitious frequencies at which unique solutions for the exterior problem cannot be obtained. However, a linear combination of Eqs. (4) and (5) will always have unique solutions [7]. That is, the following linear combination of Eqs. (4) and (5) (CHBIE) yields unique solutions at all frequencies:

$$\begin{aligned}\beta\int_S\frac{\partial^2 G(\mathbf{x,y})}{\partial n(\mathbf{x})\partial n(\mathbf{y})}\varphi(\mathbf{y})dS(\mathbf{y})+\int_S\frac{\partial G(\mathbf{x,y})}{\partial n(\mathbf{y})}\varphi(\mathbf{y})dS(\mathbf{y})+c(\mathbf{x})\varphi(\mathbf{x})-\varphi^I(\mathbf{x})\\ =\beta\left[q^I(\mathbf{x})-c(\mathbf{x})q(\mathbf{x})+\int_S\frac{\partial G(\mathbf{x,y})}{\partial n(\mathbf{x})}q(\mathbf{y})dS(\mathbf{y})\right]+\int_S G(\mathbf{x,y})q(\mathbf{y})dS(\mathbf{y}),\end{aligned} \tag{6}$$

where β is a coupling constant that must be a complex number and can be chosen, for example, as i/k. This CHBIE formulation is referred to as the Burton–Miller formulation.The acoustic problem considered in this paper is to solve Eq. (6) with the fast multipole BEM under given boundary conditions.

3. Diagonal form fast multipole method

The FMM is employed to solve the Burton–Miller BIE, or CHBIE (6), for which iterative solver GMRES will be used. Two earlier versions of the FMM are available in the literature. One is based on a multipole expansion of the kernel, named low frequency method, and another based on a plane wave expansion of the kernel, referred as the diagonal form method. Both of them have their drawbacks. It is costly and sometimes not applicable to perform low frequency fast multipole BEM in the high frequency regime. On the other hand, due to the divergence of the translations when the size of the clusters becomes very small compared with the wavelength and round-off errors of the translations, the diagonal form is unstable when it is used in the low frequency range. Despite their limitations, those methods have been proved to be very successful in their suitable frequency ranges.

The diagonal form FMM is based on a plane wave expansion of the kernel, which can be described by the following expansion [17]:

$$G(\mathbf{x,y})\approx\sum_{n=0}^{N_l}\frac{ik}{8\pi}\frac{\omega_n}{2N_l+1}\sum_{m=0}^{2N_l}I_n^m(k,\mathbf{x},\mathbf{x}_c)T_n^m(k,\mathbf{x}_c,\mathbf{y}_c)O_n^m(k,\mathbf{y}_c,\mathbf{y}), \tag{7}$$

for $|\mathbf{x}-\mathbf{x}_c|<|\mathbf{y}-\mathbf{x}_c|$ and $|\mathbf{y}-\mathbf{y}_c|<|\mathbf{x}-\mathbf{y}_c|$, where $\mathbf{x}_c$ is an expansion point near $\mathbf{x}$ and $\mathbf{y}_c$ is that near $\mathbf{y}$, N_l is the truncation number of the multipole expansion. The inner, translation and outer functions in Eq. (7) are defined by

$$I_n^m(k,\mathbf{x},\mathbf{x}_c)=e^{ik(\mathbf{x}-\mathbf{x}_c)\cdot\hat{\mathbf{s}}_{nm}}, \tag{8}$$

$$T_n^m(k,\mathbf{x}_c,\mathbf{y}_c)=\sum_{l=0}^{N_l}i^l(2l+1)h_l^{(1)}(ku)P_l(\hat{\mathbf{u}}\cdot\hat{\mathbf{s}}_{nm}), \tag{9}$$

$$O_n^m(k,\mathbf{y}_c,\mathbf{y})=e^{ik(\mathbf{y}_c-\mathbf{y})\cdot\hat{\mathbf{s}}_{nm}}, \tag{10}$$

respectively, where $u=|\mathbf{x}_c-\mathbf{y}_c|$ and $\hat{\mathbf{u}}=(\mathbf{x}_c-\mathbf{y}_c)/u$, P_l is lth order Legendre function, $\hat{\mathbf{s}}_{nm}=(\sin\theta_n\cos\varphi_m,\sin\theta_n\sin\varphi_m,\cos\theta_n)$ in which

$\varphi_m = 2\pi m/2N_l + 1$, $\theta_n = a\cos(x_n)$ and x_n, ω_n are the abscissa and weights for the Gaussian quadrature.

We define the moment of the diagonal form FMM for element j, which is far away from the collocation point $\boldsymbol{x}$ as

$$M^m_{n,j}(k,\mathbf{y}_c) = \int_{\Delta S_j} e^{ik(\mathbf{y}_c - \mathbf{y})\cdot\hat{\mathbf{s}}_{nm}} q(\mathbf{y}) dS(\mathbf{y}). \tag{11}$$

Several numerical integration schemes can be readily used to calculate the moment. However, in this paper we will present an analytical method for the moment calculation, which is more efficient, stable and accurate than the numerical integration.

The upward and downward passes in the diagonal form FMM are separated into two parts. First, in the upward pass the moments are temporarily shifted from the child level to the parent level. Conversely, in the downward pass the local expansions are temporarily shifted from the parent level to the child level. They are computed, respectively, by

$$\tilde{M}^m_n(k,\mathbf{y}_{c'}) = e^{ik(\mathbf{y}_{c'} - \mathbf{y}_c)\cdot\hat{\mathbf{s}}_{nm}} M^m_n(k,\mathbf{y}_c) \text{ for } |\mathbf{y} - \mathbf{y}_{c'}| < |\mathbf{x} - \mathbf{y}_{c'}|, \tag{12}$$

$$\tilde{L}^m_n(k,\mathbf{x}_{c'}) = e^{ik(\mathbf{x}_c - \mathbf{x}_{c'})\cdot\hat{\mathbf{s}}_{nm}} L^m_n(k,\mathbf{x}_c) \text{ for } |\mathbf{x} - \mathbf{x}_{c'}| < |\mathbf{y} - \mathbf{x}_{c'}|. \tag{13}$$

Second, the interpolations are performed over a spherical surface for the temporary moments and local expansions in the upward and downward pass, respectively. To perform the interpolation, we adopt the fast uniform resolution spherical filter method [35] in which Christoffel–Darboux formula and 1-D FMM are used to reduce the complexity. Since a none-leaf cell contains at most eight cells, in the upward pass, instead of performing interpolations for all child cells one by one, we perform the first step of all the child cells and then add all the temporary moments, and at last we perform the interpolation step. This can reduce almost half of the computing complexity for the M2M translation. A similar method is applied to the L2L translation.

In the case of 3-D domains, the maximum number of cells in the interaction list is 189. M2L translations need to be performed for all the cells in the interaction list. It is the most expensive part of the FMM algorithm. The new definition of the interaction list presented in Ref. [27] is adopted to reduce the M2L translations. To convert the moment of one cell's well-separated cell, the following equation is applied:

$$L^m_n(k,\mathbf{x}_c) = T^m_n(k,\mathbf{x}_c,\mathbf{y}_c) M^m_n(k,\mathbf{y}_c), \tag{14}$$

for $|\mathbf{x} - \mathbf{x}_c| < |\mathbf{y} - \mathbf{x}_c|$ and $|\mathbf{y} - \mathbf{y}_c| < |\mathbf{x} - \mathbf{y}_c|$. In this paper, the translation coefficients T^m_n are computed in a pre-computing process for all levels and stored for reuse in order to accelerate the computations. In the downward pass, if a leaf cell is reached, the final evaluation is computed in terms of local expansion using Eq. (7) with $\mathbf{x}$ being the collocation point in the leaf.

4. Boundary element discretization

We use N constant triangular elements to discretize the boundary S, that is, an element with three vertices where the pressure or velocity on the element is assumed to be a constant. The corresponding discretized form of the Burton–Miller BIE formulation can be written as

$$\sum_{j=1}^{N} f_{ij}\varphi_j = \sum_{j=1}^{N} g_{ij} q_j + \tilde{b}_i \text{ for } i = 1,2,\ldots,N, \tag{15}$$

where $\tilde{b}_i$ is from incident wave for scattering problem, and on element ΔS_j

$$f_{ij}\varphi_j = \left\{ \beta \int_{\Delta S_j} \frac{\partial^2 G(\mathbf{x}_i,\mathbf{y})}{\partial n(\mathbf{x}_i)\partial n(\mathbf{y})} dS(\mathbf{y}) + \int_{\Delta S_j} \frac{\partial G(\mathbf{x}_i,\mathbf{y})}{\partial n(\mathbf{y})} dS(\mathbf{y}) + \frac{1}{2}\delta_{ij} \right\} \varphi_j, \tag{16}$$

$$g_{ij} q_j = \left\{ \beta \left[\int_{\Delta S_j} \frac{\partial G(\mathbf{x}_i,\mathbf{y})}{\partial n(\mathbf{x}_i)} dS(\mathbf{y}) - \frac{1}{2}\delta_{ij} \right] + \int_{\Delta S_j} G(\mathbf{x}_i,\mathbf{y}) dS(\mathbf{y}) \right\} q_j, \tag{17}$$

with δ_{ij} being Kronecker δ-symbol. FMM is used to compute Eqs. (16) and (17) efficiently for two well separated cells. Eqs. (16) and (17) imply that for each pair of (i,j), there are four integrals that need to be evaluated. Moments for kernel $G(\mathbf{x}_i,\mathbf{y})$ is described in the previous section, and the moments for the other three kernels can be dealt with similarly. For more details, please refer to Refs. [25,34,36].

For adjacent elements, direct method is used to compute their contributions. In the implementation of the fast multipole BEM, we pre-compute and store the coefficients f_{ij} and g_{ij} of element pair (i,j) in the same cell. It can be used as preconditioner for the iterative solver (GMRES). Marked improvement can be achieved, especially when the numerical integration requires large numbers of quadrature points due to the strong variation of the kernels. The singular and hypersingular integrations in Eqs. (16) and (17) are evaluated using the singularity subtraction approach [11,25].

5. Analytical evaluation of the moments

The moment of the diagonal form fast multipole BEM for Helmholtz equation in essence is a surface integral of an exponential function on an element as is found in Eq. (11). In this section, we present analytical expressions for evaluating the moment. First, we define a translation from the x, y, z global Cartesian coordinates to the ξ_1, ξ_2, η local coordinates defined over an element, where ξ_1, ξ_2, are the oblique coordinates, which can also be treated as the area coordinates and η is the direction of the normal [34,37]. To simplify the expression, we omit the subscripts and superscript from Eq. (11). Using area coordinates, for a constant element used in the diagonal form FMM, Eq. (11) becomes

$$M(\boldsymbol{s}) = 2Aq \int_0^1 \int_0^{1-\xi_1} e^{ik(\mathbf{y}_c - \mathbf{y})\cdot\mathbf{s}} d\xi_2 d\xi_1, \tag{18}$$

where A is the area of the element, and q is the constant physical quantity on the element. The Cartesian coordinates of point $\mathbf{y}$ is related to area coordinates by

$$\mathbf{y} = \xi_1 \mathbf{y}_{13} + \xi_2 \mathbf{y}_{23} + \mathbf{y}_3, \tag{19}$$

where $\mathbf{y}_1$, $\mathbf{y}_2$, and $\mathbf{y}_3$ are the three vertices of the element, $\mathbf{y}_{13} = \mathbf{y}_1 - \mathbf{y}_3$, $\mathbf{y}_{23} = \mathbf{y}_2 - \mathbf{y}_3$. As shown in Fig. 1, $\boldsymbol{n}$ is the unit normal vector of the element. Suppose $\beta = -\boldsymbol{s}\cdot\mathbf{y}_{13}$, $\gamma = -\boldsymbol{s}\cdot\mathbf{y}_{23}$, $\rho = 2Aqe^{ik(\mathbf{y}_c - \mathbf{y}_3)\cdot\mathbf{s}}$. Integrating the integral in Eq. (18) analytically,

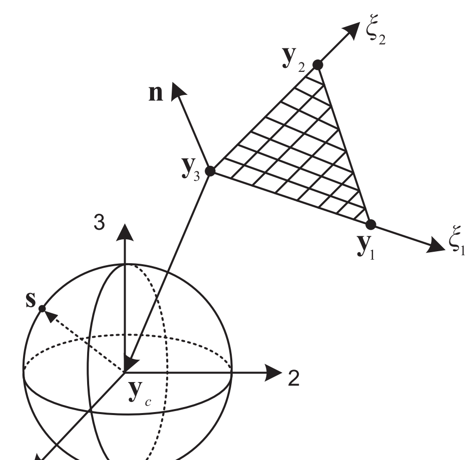

Fig. 1. Sketch of moments of an element.

Table 1
Analytical integration results for the moment.

Cases		$M(\mathbf{s})$
$\lvert \mathbf{s}\cdot\mathbf{n}\rvert=1$		$\rho/2$
$\lvert \mathbf{s}\cdot\mathbf{n}\rvert\neq 1$	$\beta=0$	$\rho(e^{\gamma}-1-\gamma)/\gamma^2$
	$\gamma=0$	$\rho(e^{\beta}-1-\beta)/\beta^2$
	$\beta-\gamma=0$	$\rho\left(e^{\gamma}-\frac{e^{\beta}-1}{\beta}\right)/\gamma$
	$\beta\gamma(\beta-\gamma)\neq 0$	$\rho\frac{1}{\gamma}\left[e^{\gamma}\frac{(e^{\beta-\gamma}-1)}{\beta-\gamma}-\frac{1}{\beta}(e^{\beta}-1)\right]$

we obtain the analytical results for the moments as shown in Table 1.

Analytical integration of the moment derived here are for constant elements only, but the approach is not limited to this. Analytical moments for linear elements are also available for the diagonal form fast multipole BEM. Moments for hypersingular kernel are different from that in Table 1 by a constant factor, $-ik\mathbf{n}(\mathbf{y})\cdot\mathbf{s}$, and replacing q in ρ with φ. Since direct evaluations are still calculated by numerical method and various series truncation are used in the diagonal form fast multipole BEM, the analytical moments do not guarantee more accurate solutions, but simply improve the efficiency of moment calculation, especially for large-scale problems.

6. Numerical examples

The diagonal form fast multipole BEM based on the Burton–Miller BIE formulation with analytical integration of moments has been implemented in a computer code. In all the examples presented below, constant triangular elements are used, for which one can use singularity subtraction approach to analytically evaluate the singular and hypersingular integrals involving the static kernels, and use analytical method (Table 1) to compute the moments. The GMRES solver will stop the iteration when the residue (relative error) is below the tolerance of 10^{-4} and left block-preconditioner is used. All the computations were done on a desktop PC with a 64-bit Intel® Core™ 2 Duo CPU and 6 GB RAM, but only one core is used in computation.

6.1. Validation of the approach

First, we use the scattering problem with a rigid sphere as the example to validate the diagonal form fast multipole BEM and the developed code. As a comparison, we take the earlier developed adaptive p^3 fast multiple BEM code [27], referred to as the original code, as a benchmark for comparison regarding the accuracy. The nondimensional wavenumber tested for this problem is $ka=20$, in which k is the wavenumber and a is the diameter of the sphere. To avoid instability of the diagonal FMM, the maximum number of elements allowed in a leaf varies with respect to the total number of elements used in the BEM model. The total number of elements (DOFs), maximum number of elements allowed in a leaf (*Maxl*), the corresponding number of tree levels (*TreLev*), and the number of iterations used (*Iters*) are listed in Table 2. The results in terms of the relative errors with respect to the solution computed by the original code and by the analytical solution are plotted in Fig. 2.

To demonstrate the improvement obtained using analytical moments in the diagonal form fast multipole BEM, the CPU time and accuracy are compared between the two diagonal fast multipole BEM codes, in which one uses the analytical moments and the other uses a numerical integration scheme. Figs. 3 and 4 show their CPU time and accuracy comparisons.

Table 2
Configuration of tree and iteration times.

DOFs	3888	14,700	30,000	50,700	120,000	307,200	607,500
Maxl	20	20	20	20	30	30	50
TreLev	4	5	6	6	7	7	7
Iters	16	14	14	14	14	14	14

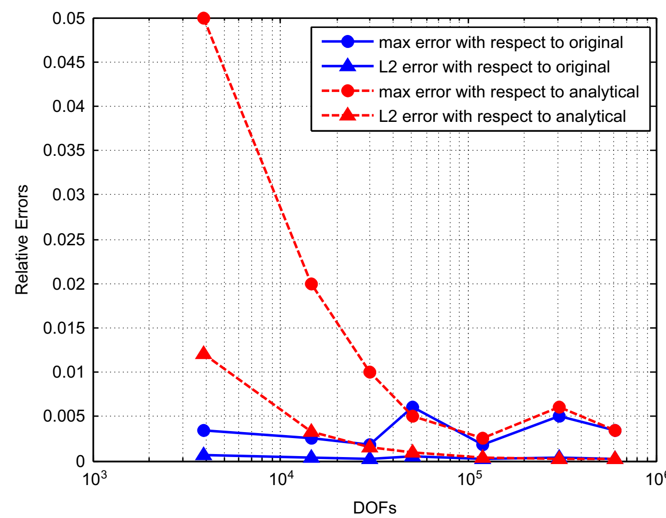

Fig. 2. Relative errors of the solutions with respect to original and analytical solution.

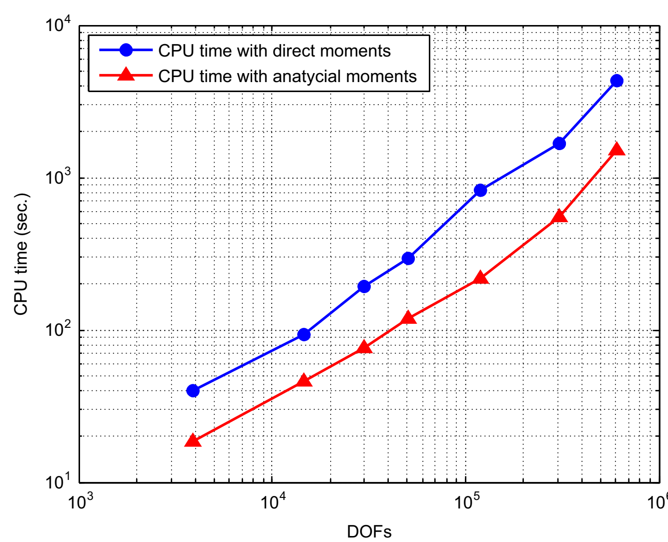

Fig. 3. CPU times used in the diagonal form fast multipole BEM solutions with the direct and analytical moments.

Fig. 2 shows that both methods have almost the same solution and the solution reaches analytical solution from 14,700 DOFs under the sense of L2 error with GMRES solver tolerance being 10^{-4}. Fig. 3 shows that analytical moments for the diagonal form fast multipole BEM can reduce about 60% CPU time if compared with direct numerical moments evaluation. Fig. 4 demonstrates that the solution, which does not use analytical moments is almost the same as that uses the analytical moments. This means that analytical moments may not guarantee more accurate solutions even if it is more accurate for each moment evaluation, but can improve the solution efficiency. These results show the accuracy of the diagonal form fast multipole BEM approach as well as its efficiency.

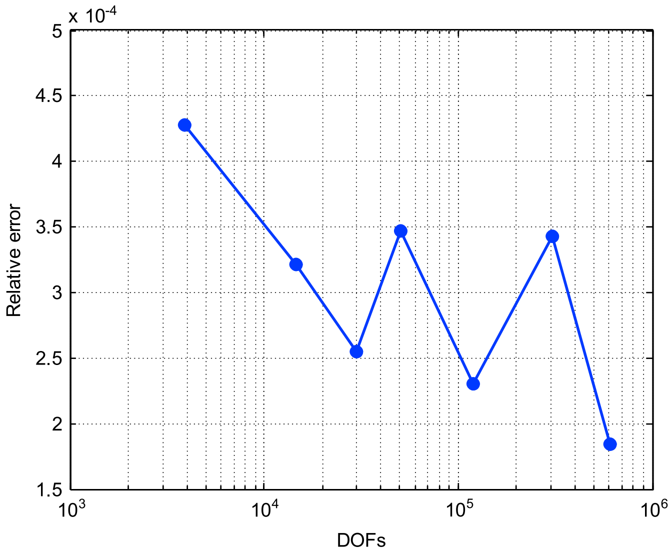

Fig. 4. Relative errors of the diagonal form fast multipole BEM solution using analytical moments with respect to that using direct moments.

6.2. Scattering from a rigid sphere

Scattering from a rigid sphere is used to test the program for solving the scattering problems at their fictitious eigenfrequency. The sphere is centered at (0, 0, 0), impinged upon by an incident plane wave with unit magnitude, traveling along positive z axis. In this case, ka is chosen as one of its characteristic frequencies, 2π. The model is meshed with 19,200 triangular elements. Field points are evenly distributed on a circle in x–z plane as shown in Fig. 5(a), and θ is the inclination angle of points in spherical coordinate system. Results of analytical, CHBIE and CBIE solutions are plotted in Fig. 5(b), which shows that the diagonal form fast multipole BEM based on the Burton–Miller BIE formulation can overcome the non-uniqueness difficulties and yield accurate results.

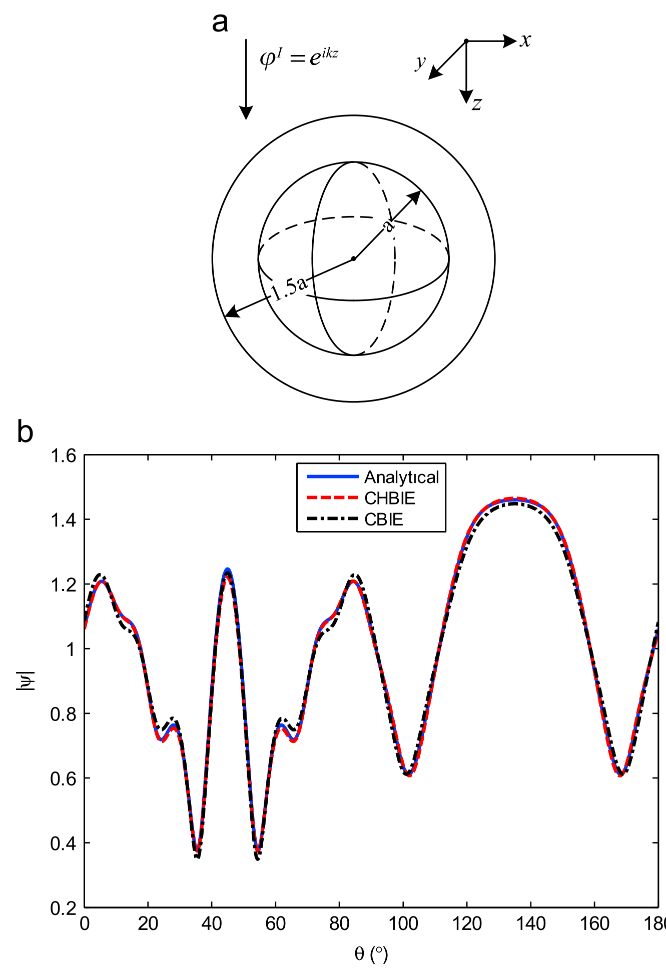

Fig. 5. Scattering from a rigid sphere at fictitious eigenfrequency $ka=2\pi$

6.3. Scattering from a dolphin model

The analysis of scattering from a dolphin model is presented next. The dolphin model is 1.9 m in length, 0.5 m in width and 0.6 m in height and the nondimensional wavenumber ka used is 114.3. The model is meshed with 404,422 elements and the incident wave is a plane wave with unit magnitude propagating in the $+x$ direction. The maximum allowed number of elements in a leaf is set as 20, and 9 levels are generated for the tree. CPU times used for solutions with analytical and direct numerical methods to compute moments are 4,137 s and 7,309 s, respectively. 43.4% CPU time is reduced in solution with analytical method to compute moments for this case. As shown in Fig. 6, near the head of the dolphin, the sound pressure registered the largest value, which makes sense since this is the part of the dolphin to generate and receive sounds under the water.

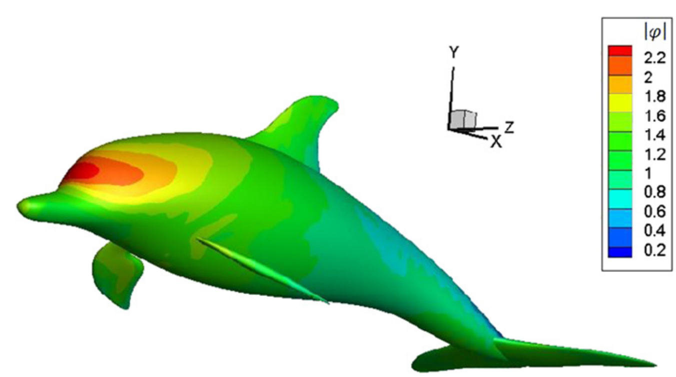

Fig. 6. Scattering from a dolphin model.

6.4. Radiation from locomotive interacting with the rail

When a train is moving with a low speed, one of the dominated noises of the train is the wheel–rail noise. We apply the diagonal form fast multipole BEM to analyze this large-scale locomotive radiation problem. The locomotive has an overall dimension of 18.4 m × 3.4 m × 5.9 m in the x, y and z direction, respectively, and is meshed with 257,972 constant elements. The model considered here is a simplified one. The sources are from eight harmonic vibrating wheels. The first step is to compute by FEM the harmonic wheel vibrations along the z direction, corresponding to a harmonic force equal to 104 N at a frequency equal to 160 Hz, applied at the wheel–rail intersection line. The velocity of the elements on the wheel surface computed by the FEM is copied to the other seven wheels and the rest of the locomotive surface is treated as rigid boundary. Then the BEM model is solved with the diagonal form fast multipole BEM with the maximum allowed number of elements in a leaf set at 20. The corresponding ka of this model is 55.78. For this complicated shape model, the total CPU time used for solutions with analytical and direct numerical methods to compute moments are 3269 s and 4072 s, respectively. 35.7% CPU time is reduced with respect to that using direct numerical method to compute moments.

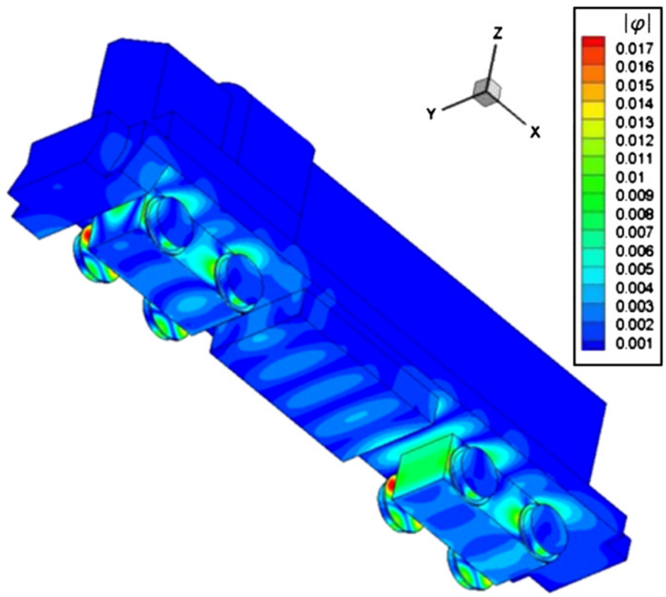

Fig. 7. Locomotive radiation.

Fig. 7 shows the computed pressure on the boundary caused by the eight vibrating wheels.

7. Conclusion

A diagonal form fast multipole BEM based on Burton–Miller BIE formulation with the new adaptive tree algorithm is presented in this paper. Analytical evaluation of the moments for diagonal form fast multipole BEM are proposed to compute the moments accurately and efficiently. The diagonal form fast multipole BEM with the analytical moments is compared with that using numerical integration, and it is demonstrated that the new approach can reduce the CPU time by about 60% for the rigid sphere scattering case, by 43.4% for the dolphin scattering case and by 35.7% for the locomotive radiation case. The differentiation in CPU time reduction largely depends on the proportion of CPU time for moments calculation to the overall solution time. Comparison of the accuracies shows that both of algorithms can converge to the analytical solution with the tolerance set at 10^{-4} for the iterative solver. Successful solutions of the BEM models with the diagonal form fast multipole BEM for the dolphin and locomotive models clearly demonstrate the potential of the approach in solving large-scale acoustic wave problems. The developed diagonal form fast multipole BEM can be readily extended to half-space acoustic problems similar to that in Ref. [26] and other acoustic problems.

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